This exam contains 20 multiple choice questions. Each question is worth 5 points.

1. Find the arc length of
$$y = \frac{x^3}{12} + \frac{1}{x}$$
over the interval $[1, 3]$. (Hint: Use the identity $1 + (y')^2 = (x^2/4 + x^{-2})^2$.)

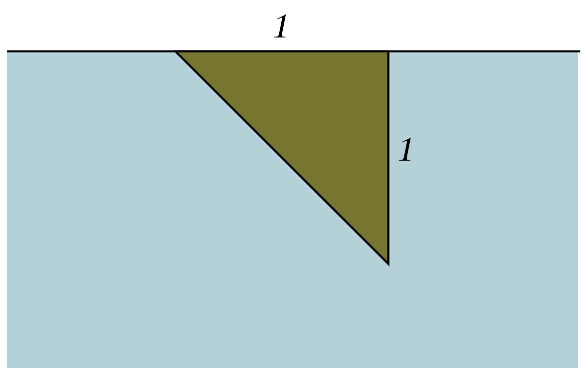
   A) 8/13
   B) 13/12
   C) 7/9
   D) 5/12
   E) 2/5
   F) 13/6
   *G) 17/6
   H) 15/12
   I) 5/6
   J) 17/12

2. Which of the following integrals correctly represents the surface area of revolution of $y = e^x$ about the x-axis over the interval $[0, 1]$?

    A) $\int_0^1 u\sqrt{1+u^2}\,du$
    B) $\int_0^1 \sqrt{1+u^2}\,du$
    C) $\pi \int_1^e u\sqrt{1+u^2}\,du$
    D) $\pi \int_1^e \sqrt{1+u^2}\,du$
    E) $2\pi \int_0^1 u\sqrt{1+u^2}\,du$
    F) $2\pi \int_0^1 \sqrt{1+u^2}\,du$
    G) $2\pi \int_1^e u\sqrt{1+u^2}\,du$
    *H) $2\pi \int_1^e \sqrt{1+u^2}\,du$
    I) $\int_1^e u\sqrt{1+u^2}\,du$
    J) $\int_1^e \sqrt{1+u^2}\,du$

3. A thin triangular plate is submerged vertically in water so that one side is level with the water's surface. The plate is a right-triangle with two sides of length 1. Let w be the weight density of water. Calculate the force of the water on the surface of the plate.

*A) $w/6$
B) $w/5$
C) $w/4$
D) $w/3$
E) $w/2$
F) w
G) $2w/3$
H) $2w/5$
I) $2w/7$
J) $3w/5$



4. Calculate the Taylor polynomial $T_2(x)$ at $x = 1$ for $f(x) = \ln x$.

A) $(x-1)-(x-1)^2/6$

B) $x+x^2/3$

C) $x-x^2/2$

D) $x+x^2/6$

E) $x+x^2/2$

F) $(x-1)-2(x-1)^2$

G) $(x-1)-(x-1)^2$

*H) $(x-1)-(x-1)^2/2$

I) $x+2x^2$

J) $(x-1)+(x-1)^2/2$

5. Use the error bound formula to find an upper bound on the error

$$|f(1.5) - T_3(1.5)|$$

in approximating $f(1.5)$ by its Taylor polynomial centered at $x = 1$. Assume that $|f^{(4)}(u)| \leq 2$ for all u between 1 and 1.5.

A) 1/269
B) 1/136
*C) 1/192
D) 1/428
E) 1/320
F) 1/284
G) 1/216
H) 1/36
I) 1/196
J) 1/96

6. Solve the initial value problem:

$$yy' = xe^{-y^2}, \quad y(0) = 1.$$

A) $y = \sqrt{\ln(x^2)}$
B) $y = \cosh(x^2) + 2x + 1$
C) $y = \ln(\sqrt{x^2 + e})$
D) $y = \ln(\sqrt{x^2 + e})$
*E) $y = \sqrt{\ln(x^2 + e)}$
F) $y = e^{x^2} + 2x + C$
G) $y = \frac{1}{2}\ln(\sqrt{x^2 + e})$
H) $y = \frac{1}{2}\ln(x^2 + e)$
I) $y = e^{-x^2} + C$
J) $y = \frac{1}{2}\ln(x^2)$

7. A cup of coffee, cooling off in a room at temperature $30\,^\circ\mathrm{C}$, has cooling constant $k = 0.08\ \mathrm{min}^{-1}$. If the coffee is served at a temperature of $90\,^\circ\mathrm{C}$, how long should you wait until its temperature drops to $60\,^\circ\mathrm{C}$?

   A) 15.2 min
   B) 20.3 min
   C) 3.9 min
   D) 10.2 min
   E) 16.6 min
   *F) 8.7 min
   G) 4.8 min
   H) 5.1 min
   I) 2.7 min
   J) 12.5 min

8. Find the solution of the initial value problem:

$$y' = 3y\left(1 - \frac{y}{4}\right), \quad y(0) = 2.$$

*A) $y = 4/(1 + e^{-3t})$
B) $y = 4/(1 - e^{-3t})$
C) $y = 2/(1 + e^{-3t})$
D) $y = 2/(1 - e^{-3t})$
E) $y = 4/(1 + e^{-4t})$
F) $y = 2/(1 + 2e^{-2t})$
G) $y = 3/(1 + 4e^{-2t})$
H) $y = 2/(1 - 4e^{-3t})$
I) $y = 2/(1 + 4e^{-3t})$
J) $y = 4/(1 + e^{-2t})$

9. Find the solution of the initial value problem:

$$y' + 3y = e^{-3x}, \quad y(0) = -1.$$

A) $(x+1)e^{-3x}$
B) $(1-x)e^{-3x}$
C) $(2x-1)e^{-3x}$
D) $xe^{-3x} + e^{3x}$
*E) $(x-1)e^{-3x}$
F) $e^{-3x} + e^{3x}$
G) $e^{-3x} - e^{3x}$
H) $e^{-3x} + xe^{3x}$
I) $(x-1)e^{3x}$
J) $(1-x)e^{3x}$

10. A 200-gal tank contains 100 gal of water with a salt concentration of 0.1 lb/gal. Water with a salt concentration of 0.4 lb/gal flows into the tank at a rate of 20 gal/min. The fluid is mixed instantaneously, and water in pumped out at the same rate it flows into the tank. What is the limiting salt concentration for large t?

    A) 31.1 lb/gal
    B) 1.3 lb/gal
    C) 0.9 lb/gal
    D) 7.8 lb/gal
    E) 45.7 lb/gal
    F) 0.6 lb/gal
    G) 0.5 lb/gal
    *H) 0.4 lb/gal
    I) 0.3 lb/gal
    J) 32.2 lb/gal

11. Find the term a_4 of the sequence defined recursively by the equations: $a_0 = 0$, $a_1 = 1$, $a_n = a_{n-1} + a_{n-2}$ for $n \geq 2$.

A) 12

B) 11

C) 10

D) 9

E) 8

F) 7

G) 6

H) 5

I) 4

*J) 3

12. Let the n-th term of a sequence be defined by

$$a_n = \frac{n}{n+1}.$$

Find the smallest number M such that $|a_n - 1| \leq 0.0001$ for all $n \geq M$.

A) 2000
B) 200
C) 20
D) 1000000
E) 100000
F) 10000
G) 99999
*H) 9999
I) 999
J) 99

13. Find the limit

$$\lim_{n\to\infty}\left(1+\frac{2}{n}\right)^n.$$

A) e

*B) e^2

C) e^{-1}

D) e^{-2}

E) 0

F) 1

G) 2

H) πn

I) n

J) $e^{2/n}$

14. Consider the two series

$$(a) \sum_{n=2}^{\infty} \left(\frac{1}{\ln n} - \frac{1}{\ln(n+1)} \right), \quad (b) \sum_{1}^{\infty} \left(\sqrt{n} - \sqrt{n+1} \right).$$

Do these series converge? If they do, what is their value?

A) (a) converges to $1/\ln 2$; (b) converges to 1

*B) (a) converges to $1/\ln 2$; (b) diverges

C) (a) converges to 0; (b) diverges

D) (a) diverges; (b) converges to 1

E) (a) diverges; (b) diverges

F) (a) converges to $1/\ln 2$; (b) converges to 0

G) (a) converges to $1/2$; (b) diverges

H) (a) converges to $1/2$; (b) converges to $\sqrt{2}$

I) (a) diverges; (b) converges to $\sqrt{2}$

J) (a) diverges; (b) converges to $\sqrt{5}$

15. Find the value of the series

$$\sum_{n=3}^{\infty} e^{3-2n}.$$

*A) $e^{-3}/(1-e^{-2})$
B) $e/(e^2-1)$
C) $e^{-2}/(1-e^{-3})$
D) $e/(e^3-2)$
E) $e^{-1}/(1-e^{-3})$
F) $e^3/(e^2-1)$
G) $1/(1-e^{-2})$
H) $1/(e^2-1)$
I) $3/(1-e^{-2})$
J) The series diverges

16. Determine whether the series

$$(a)\sum_{n=1}^{\infty} n^{-1/3}, \quad (b)\sum_{n=3}^{\infty} \frac{n^2}{(n^3+9)^{5/2}}, \quad (c)\sum_{n=1}^{\infty} \frac{2}{n+\sqrt{n}}, \quad (d)\sum_{n=1}^{\infty} \frac{\sin^2 k}{k^2}$$

converge or diverge. (c = converges and d = diverges.)

A) c, d, c, d
B) c, c, d, d
C) d, d, c, c
D) d, c, d, d
E) d, c, c, d
*F) d, c, d, c
G) c, d, d, c
H) c, d, d, d
I) d, d, d, c
J) d, d, d, d

17. Using the error formula for alternating series, what error do you make by approximating the sum

$$S = \sum_{n=1}^{\infty} \frac{(-1)^n}{n(n+2)(n+3)}$$

by S_3 (i.e., the sum of the first three terms)?

A) 1/10
B) 1/90
C) 1/100
*D) 1/168
E) 1/24
F) 1/356
G) 1/1000
H) 1/10000
I) 1/500
J) 1/350

18. For the following two series, determine the range of values of x for which each series converges.

$$(a) \sum_{n=1}^{\infty} 3^n x^n; \quad (b) \sum_{n=1}^{\infty} \frac{x^n}{n}.$$

A) (a) $|x| < 3$; (b) $|x| < 4$

B) (a) $|x| < 3$; (b) $|x| < 3$

C) (a) $|x| < 3$; (b) $|x| < 2$

D) (a) $|x| < 3$; (b) $|x| < 1$

E) (a) $|x| < 1/3$; (b) converges for all x

*F) (a) $|x| < 1/3$; (b) $|x| < 1$

G) (a) $|x| < 1/2$; (b) $|x| < 2$

H) (a) $|x| < 1$; (b) $|x| < 3$

I) (a) converges for all x; (b) $|x| < 1$

J) (a) and (b) converge

19. Find the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{x^n}{ne^n}.$$

A) 1

B) ∞

C) 2

*D) e

E) e^2

F) $2e$

G) $3e$

H) $e/2$

I) 4

J) 0

20. The function $f(x) = 1/(1+x^9)$ can be expanded as a power series of the form

$$\sum_{n=0}^{\infty}(-1)^n x^{9n}.$$

For what values of x is the expansion valid?

A) for all x

B) $|x| < 9$

C) $|x| < 3$

D) $|x| < 2$

E) $|x| > 1$

F) $|x| > 2$

G) $|x| > 9$

*H) $|x| < 1$

I) $|x| = 1$

J) the series diverges for all x