Math 450 - Homework 4

Due date: Friday, 2/16/07

1. Read the introduction and sections 1.1, 1.2, 1.3 and 1.4 of the first chapter of Norris' book.

2. Go back to our `samplefrommarkov` program, think about how it works, then read Exercise 1.1.3, page 8, of Norris' book. See if you can explain the logic of the program in the language of the exercise (i.e., using the function $G : I \times [0,1] \to I$. What is I and how do you write the function G in terms of the transition probability matrix?) There is nothing to write-up in this problem, but as you will be simulating Markov chains later, it is helpful to become familiar now with how it is done. You may prefer to write your own program, in which case Exercise 1.1.3 suggests a nice approach.

3. A house has 9 rooms as shown in figure 1. In room 7 sits a cat and in room 9 there is a hole in the wall where a mouse can hide. A mouse moves around the house choosing the doors at random, starting at room 3. The movement ends when the mouse enters room 7 or 9. (Rooms 7 and 9 are "absorbing states" of a Markov chain.)

   (a) Draw a directed graph representing a Markov chain for the mouse's random walk. (E.g., there are transitions from 5 to 2, 4, 6, and 8, with probabilities 1/4. Room 6, on the other hand, has only three doors so the transitions from it have probabilities 1/3.)

   (b) Write down the probability transition matrix P.

   (c) Let $u_7(n)$ (respectively, $u_9(n)$) represent the probability that the mouse will be visiting room 7 (respectively, 9) at the nth room change. On a same coordinate system, plot the graphs of $u_7(n)$ and $u_9(n)$ for n from 1 to 50. (Note: these numbers can be obtained from the powers P^n.) Also find approximate values for

   $$\lim_{n \to \infty} u_7(n) \quad \text{and} \quad \lim_{n \to \infty} u_7(n).$$

   (d) From what you obtain in the previous item, explain why the following is true: after long enough, the mouse will either be with the cat or in the wall hole, with probabilities p and $1 - p$. What is p? (It is not

difficult to obtain the exact value of p, and we will see how to do it later. For now, simply state the value suggested by the graph.)

(e) Write a program to simulate the mouse's random walk. Simulate 1000 runs of the chain, starting from state 3, and calculate the proportion of trials where the mouse arrives at 9 without going into 7.
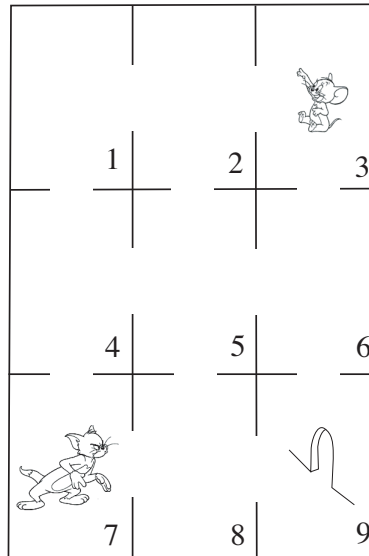

Figure 1:

4. Do Exercise 1.2.1 on page 11 of Norris' book.