Math 450 - Homework 10

Due date: Friday, 4/06/07

1. Read lecture notes on Petri nets, as well as sections 3.5 and 3.6 of textbook.

2. (This is about problem 3.7.1 of textbook. The notions of reaction rate is in the on-line notes.) Consider a fleet of N buses. Each bus breaks down independently at rate μ and is then sent to the depot for repair. The repair shop can only repair one bus at a time and each bus takes an exponential time of parameter λ to repair.

   (a) Draw a Petri net diagram of this situation. (Note: there are two places: in service buses and out of order buses; and two transitions: bus-breaks-down (in-service $\rightarrow$ out-of-order) and bus-is-repaired (the reverse reaction). Indicate on the diagram the reaction rates. Is the mass-action law applicable to both reactions in this example?)

   (b) Suppose that $N = 4$. Draw the complete state transition diagram (see section 2, page 4, of the notes on Petri nets) indicating the transition rates q_{ij} in terms of μ and λ.

   (c) Obtain analytically the equilibrium distribution of the number of buses in service. (This is the original question in the book.) What is the expected value of the number of buses in service?

   (d) Confirm your answer of the expected number of buses in service by a computer simulation. Assume that $\mu = 1/5$, $\lambda = 1$, and $N = 50$. (Choose any other simulation parameter, such as the number of trials, as you see fit.)

3. (This refers to problem 3.3.1 of textbook.) In our previous homework assignment, you obtained the expected time to hit state 4 starting from 1. Confirm your answer by a computer simulation of the process. Choose any simulation parameter as you see fit.

4. Do the problem of section 4 of the notes. (This is about the stochastic predator-prey model with migration.)