Math 308 - Exam 3 - April 11, 2003

The test contains 8 questions, each of equal value. Whenever possible, answers should be written using exact numbers. For example: write $\frac{2}{3}$ instead of 0.67, π instead of 3.1415, e^2 instead of 7.4, etc.

1. Suppose that a given function $f(x)$ has Fourier series

$$\frac{1}{2}+\frac{1}{i\pi}\left(\frac{e^{ix}}{1}+\frac{e^{i3x}}{3}+\frac{e^{i5x}}{5}+\cdots\right)+\frac{1}{i\pi}\left(\frac{e^{-ix}}{-1}+\frac{e^{-i3x}}{-3}+\frac{e^{-i5x}}{-5}+\cdots\right).$$

Express this as a real-valued Fourier series using sine/cosine terms.

2. Let $f(x)$ be a periodic function of period 2π such that

$$f(x)=e^x, \quad -\pi<x<\pi.$$

Determine the coefficient c_2 of the Fourier series $f(x)=\sum_{n=-\infty}^{\infty} c_n e^{inx}$. Write c_2 in the form $a+ib$.

3. Let $f(x)$ be a periodic function of period $2l$ defined by

$$f(x)=\begin{cases}-1, & -l<x<0\\ 1, & 0<x<l.\end{cases}$$

Find the Fourier series of F. (Write the first 3 non-zero terms of the sine/cosine series.)

4. Using Leibniz' rule, find the derivative

$$\frac{d^{100}}{dx^{100}}\left(x^2e^{-x}\right).$$

5. Express the polynomial x^2+3x+1 as a linear combination of the Legendre polynomials:

$$P_0(x)=1,\ P_1(x)=x,\ P_2(x)=\frac{1}{2}(3x^2-1),\ P_3(x)=\frac{1}{2}(5x^3-3x),\ \ldots.$$

6. Find the Legendre series $c_0P_0(x)+c_1P_1(x)+c_2P_2(x)+\cdots$

$$f(x)=\begin{cases}0, & -1<x<0\\ x, & 0<x<1.\end{cases}$$

up to degree 3.

7. If $f(x)$ is the function given in Problem 6, sketch the graph of the function

$$g(x)=1-f(x)-f(-x).$$

8. Expand the function $g(x)$ of Problem 7 in Legendre series up to third degree terms.