Other Logical Connectives Used in English

English has many connectives to use for joining propositions together. The simplest are: "not" ($\sim$), "and" ($\wedge$), "or" ($\vee$), "implies" ($\Rightarrow$), "is equivalent to" ($\Leftrightarrow$). We wrote down truth tables to see exactly what each connective is intended to mean.

Another connective that we use fairly often in mathematics is "neither/nor." Here the notation is a little less standard, but $\downarrow$ is frequently used: $P \downarrow Q$. This connective $\downarrow$ is defined to have the same truth table as $\sim P \wedge \sim Q$ (or, equivalently, as $\sim (P \vee Q)$).

The connective "not both P and Q" is also used sometimes. It is written $P|Q$ (or $P \uparrow Q$ in some books). It has the same truth table as $\sim (P \wedge Q)$ (or equivalently, as $\sim P \vee \sim Q$).

In this course, we will usually get by with $\sim$, $\wedge$, $\vee$, $\Rightarrow$, and $\Leftrightarrow$. But, at least, you should also know how to write "neither/nor" in terms of these.

Actually, all possible logical connectives one could invent have an equivalent form that can be written in terms of $\sim$, $\wedge$, $\vee$, $\Rightarrow$, and $\Leftrightarrow$. This is a theorem from a course in propositional logic. So, in some sense, we know we haven't short-changed ourselves by using just these.

In fact, we could economize even more if we wanted to. There's a lot of redundancy in the connectives. If our interest were just to minimize the number of connectives used, we could (for example) completely avoid using $\wedge$: whenever we want to say "$P \wedge Q$" we could just substitute the equivalent form $\sim (\sim P \vee \sim Q)$. But this economy would come at the price of making expressions harder to read.

To take economy to the extreme, it's <u>possible</u> to express all connectives in terms <u>just one</u> connective: for example, any propositional form using the connectives $\sim$, $\wedge$, $\vee$, $\Rightarrow$, and $\Leftrightarrow$ can be replaced by an equivalent form that uses only "neither/nor" $\downarrow$.

For example, $\sim P$ is equivalent to $P \downarrow P$ ("neither P nor P"), and

$P \wedge Q$ is equivalent to $(\sim P) \downarrow (\sim Q)$

Try writing $\vee$, $\Rightarrow$, $\Leftrightarrow$ in terms of $\downarrow$.

Certain connectives such as $\downarrow$ are special: for example, you <u>cannot</u> write $\sim P$ in an equivalent form that uses only $\wedge$.

There are other connective words in English that (usually) have the same <u>logical</u> meaning as "and": these connectives include "but", "while", "although." For example:

<u>John likes spaghetti</u> but <u>John does not like macaroni.</u>
    ↑   ↑
    P Q

This sentence is true when P and Q are true, and false otherwise – so it is <u>logically equivalent</u> to $P \wedge Q$.

> *Although this sentence has the same <u>truth value</u> whether we use "but" or "and", the use of "but" gives the sentence a different tone. Here "but" suggests an unexpected contrast – because spaghetti and macaroni are such similar foods. In English, someone would probably say "John likes spaghetti <u>and</u> John doesn't like Clint Eastwood" instead of "John likes spaghetti <u>but</u> John doesn't like Clint Eastwood" – even though there's no logical difference.*

Sometimes English sentences use connectives in ways that make it unclear how to write the meaning of the connective. This happens when a connective in an English proposition is ambiguous; a decision about the English meaning is needed before you can decide what logical connective is being used.

Example In this example, the meaning of the English connective <u>unless</u> depends on the "story-line." John and Jane are considering whether to go to Blueberry Hill (BBH) for dinner.

> <u>Story 1</u>) John is reluctant to go out. The conversation starts:
>
>> John: I won't go to BBH unless BBH has chili.
>> Jane: BBH has chili.
>> John: I also won't go to BBH unless BBH has champagne.
>> Jane: BBH has champagne too.
>>   etc.
>
> Let $P =$ "I will go to BBH"
> $Q =$ "BBH has chili"
> $R =$ "BBH has champagne"
>
> The connective "unless" in John's first statement means that chili is <u>necessary</u> for going to BBH but chili <u>not sufficient</u> – because John wants other things too, like champagne. In this story,
>
>> John's first statement says $P \Rightarrow Q$ (Q <u>is necessary for</u> P)
>> but it <u>doesn't</u> say $Q \Rightarrow P$ (Q <u>is sufficient for</u> P)
>
> <u>Story 2</u>) John loves going to BBH and really wants to go there with Jane for their

birthday dinner. His only worry is whether champagne will be available for the celebration: if BBH has it, that's where they'll go for sure.

John says: I won't go to BBH unless BBH has champagne

In the context of this story, "unless" means that champagne is <u>necessary and sufficient</u> to get John to go to BBH. In this context, John is saying $P \Leftrightarrow R$.

<u>Exercise</u>: Try to invention a variation of this story where "unless" means that chili (or champagne) is sufficient but not necessary.