132 Exam I Special Review Problems

These six problems are designed to be deeper and more comprehensive than homework. We do not intend for you to study just this sheet in preparation for Exam I. In fact, you may want to work on these only after you feel prepared for the exam. Enjoy!

1) Find a function f, an interval $[a, b]$, and a value of $n \geq 2$ such that the midpoint approximation of $\int_a^b f(x)\,dx$ with n subintervals gives an overestimate. An underestimate? Exactly the right answer?

2) For what value of $a \leq 2$ is $\int_a^2 2 + x - x^2\,dx$ greatest?

3) The two integrals $\int_0^1 2xe^x\,dx$ and $\int_0^{\pi/2} e^{\sin x} \sin 2x\,dx$ are equal. Explain why this is **not** a coincidence.

4) Evaluate

$$\lim_{x\to 3}\left[\lim_{n\to\infty} \frac{1}{x-3}\sum_{i=1}^{n}\left(3+\frac{i(x-3)}{n}\right)^2\left(\frac{x-3}{n}\right)\right].$$

5) Suppose f is continuous, $f(0) = 0$, $f(1) = 1$, $f'(x) > 0$, and $\int_0^1 f(x)\,dx = 1/3$. Find $\int_0^1 f^{-1}(y)\,dy$.

6) Evaluate $\int_0^1 x\sqrt{1-x^4}\,dx$.