Review Problems on Derivatives

(Brief solutions are posted in the web syllabus together with the daily assignment for the first day of class.)

1. Find the derivative $\frac{dy}{dx}$ ($=f\,'(x)$)

a) $y = f(x) = \sin(\cos(\tan x))$

b) $y = f(x) = \ln(\sec^2(x^2+1))$

c) $y = f(x) = 10^{\arctan(2^x)}$

d) $y = f(x) = \log_7(xe^{\sqrt[3]{x^2+1}})$

e) $y = f(x) = x^{\arcsin(x^2)}$

f) $y = f(x) = \frac{\arctan x}{1+x^2}$

g) $y = f(x) = \ln|\sin x|$

h) $x^2y^2 + \sin y = x$

i) $x^y = y^x$

2. The Mean Value Theorem (see text, pp. 280-282 for a review) states:

If f is a differentiable function on the interval $[a, b]$, then there exists a number c between a and b such that

(*) $f\,'(c) = \frac{f(b)-f(a)}{b-a}$ (or, equivalently, $f\,'(c)(b-a) = f(b) - f(a)$

The Mean Value Theorem is very important – not so much for computations as a tool in understanding other useful tools and facts in calculus.

a) Draw a picture of a continuous function with domain some interval $[a, b]$. Draw the straight line segment L joining the points $(a, f(a))$ and $(b, f(b))$ in your picture. What is its slope (in terms of $a, b,$ and f) ?

b) $f\,'(c)$ represents the slope the tangent line to the graph at the point $(c, f(c))$. Find a point in your picture where the equation (*) is true. What does the equation say geometrically?

c) Suppose we have a car moving along a straight highway. Its position (in km) at time x hours is given by the function $y = f(x)$. What does the right hand side of equation (*) represent physically? what does the left hand side represent physically?

d) Suppose you drive from here to Kansas City along a straight line highway – more or less like I-70. Your average velocity is 100 km/hr. What does the Mean Value Theorem tell you happened at some time c during the trip?

3. Suppose we have a function $y = f(x)$ for which

$$f'(x) = (x-1)(x-2)^2$$

a) Where is the graph of $f(x)$ increasing? decreasing?
b) Where does $f(x)$ have a local maximum or minimum?
c) Where is the graph of y $f'(x)$ increasing? decreasing?
d) Where is the graph of $f(x)$ concave up? concave down?
e) Where does $f(x)$ have inflection points?

4. The table bleow gives the position $s = f(t)$ (in feet) of a point moving along a line after t seconds.

t	0	1	2	3
s	0	3	4	6

a) Based on this data, what is the best <u>estimate</u> you can make of the velocity of the point at time $t = 2$?

b) The answer to a) is your estimate to the value of what derivative?