HOMEWORK 2, MATH 233
DUE TUESDAY, JANUARY 22, 2002

NAME _____

In every problem explain what you are doing in each important step. Details of calculations should not usually be included. An exception to that might be a calculation which requires a special insight or trick. Think of communicating with the reader as you write up your homework solutions. Usually, you should work out a problem first on scratch paper, then write up the solution to hand in.

Problems 1 and 3 are worth 8 points apiece, problems 2 and 4 are worth 7 points apiece. Total: 30 points.

(1) See Exercise 24 on page 660 of §9.2 for more details. Suppose that a wind is blowing from the direction $N45°W$ at a speed of 50 km/h. A pilot is steering a plane in the direction $N60°E$ at an airspeed (speed in still air) of 250 km/h.
   (a) Find the true course, expressed as a vector, of the plane.
   (b) Find the ground speed of the plane.
   (c) Express the direction of the true course in the form $N\theta°E$.
   (d) Sketch by hand a vector showing the wind velocity, a vector showing the steered (or virtual) velocity of the plane and a vector showing the true velocity of the plane.

(2) (#36 on page 667 of §9.3). Find the angle between a diagonal of a cube and a diagonal of one of its faces.

(3) (Read §6.1 of ML up to §6.1.1, pages 101 - 102). Consider the points in space $P(0, -1, 6)$, $Q(2, 1, -3)$ and $R(5, 4, 2)$.
   (a) Use MATLAB to plot these three points and join them to form a triangle. Here is a sample script to do this. Notice that the components of the first point, P, make up the first entries of the arrays x, y and z, respectively, in accordance with the way the plot3 command works.
   ```
   x = [0 2 5 0]; y = [-1 1 4 -1]; z = [6 -3 2 6];
   plot3(x,y,z); grid on;
   ```
   Remember to include a title and to label the axes.
   (b) Print the graph and label the three points by hand. Or, continue your script to use MATLAB to do this and give yourself some practice using the text command. Here is how to have MATLAB label the point P. Notice how the y-coordinate of its placement is shifted away from -1 so that the letter P is not printed right on top of the point itself.

`text(0,-.8,6,'P');`
Print the graph.

(c) Find the coordinates (x, y, z) of the point F which is the foot of the altitude from P to the opposite side. Hint: $\mathbf{QF} = \text{proj}_{\mathbf{QR}}\mathbf{QP}$. By hand, plot this point on your graph. Summarize your calculation in the top margin of the graph.

(d) Calculate (by hand) the area of the triangle from the formula: area equals half the base times the height $= |\mathbf{QR}||\mathbf{PF}|/2$. Compare this to what you get from the formula for the area of the triangle in terms of the vector cross product $\mathbf{QR} \times \mathbf{QP}$. Summarize these calculations in the bottom margin of your graph.

(4) (Similar to Exercise 1 on page 105 of ML). The object of this exercise is to use MATLAB to set up a 3-d grid on which it has plotted the three terminal points of vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$. On that you will draw with a ruler the edges of the parallelepiped defined by these three vectors when they are represented by arrows based at the origin. As part of your script you will use MATLAB's determinant command *det* (see §6.1.8 p.105) or cross product command *cross* (see §6.1.6 p.104) to calculate the volume of this parallelepiped and report the value in the title.

(a) Use MATLAB to plot the terminal points of the following vectors if they are based at the origin: $\mathbf{a} = \langle 2, 1, 0\rangle$, $\mathbf{b} = \langle 1, 2, 0\rangle$, $\mathbf{c} = \langle 1, 1, 3\rangle$ and $\mathbf{a} \times \mathbf{b}$. Designate each point with the mark '.' (enter *help plot3* to read about marks). For example, the first point will be plotted by plot3(2,1,0,'.'). There must be a plot3 command for each point. Use *hold on* to get all three points on the same graph. To have room to sketch the parallelepiped by hand, you will need to shrink the figure by enlarging the axis window. The following line in the script will accomplish that.
`axis([0 4 0 4 0 4]);`

(b) Include in your script a calculation of the volume of the parallelepiped determined by $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ (see §6.1.6 on p.104 or §6.1.8 on p. 105 of ML). Report the value of this volume in the title, by first creating a character array T as is done at the top of page 104 of ML. Namely, if v is the name of the variable containing the value MATLAB calculates for the volume, then your script would include the lines

```
T = ['My Name, Volume = ' num2str(v)];
title(T)
```

Print your graph.

(c) Use a straight edge to connect the points to make the parallelepiped. The vertices of the parallelepiped are $\mathbf{a} + \mathbf{b}$, $\mathbf{a} + \mathbf{c}$, $\mathbf{b} + \mathbf{c}$ and $\mathbf{a} + \mathbf{b} + \mathbf{c}$. Label these points (i.e., the heads of these vectors) by hand.