Homework 4, Math 233
Due Monday, February 4, 2002

Name ____

The point values of the problems are 8, 7, 8, 7, respectively, for a total of 30 points.

1. (§H1) The standard form for the equation of an ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where a and b are positive constants, called the horizontal and vertical semiaxes, respectively. In this form the center of the ellipse is at the origin $(0, 0)$. If the ellipse is translated so that its center becomes the point (h, k), then its equation is

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

   (a) Find the Cartesian equation for the polar equation $r = c/(a + e\cos\theta)$, where c and e are given constants satisfying $c > 0$ and $0 < e < 1$, and compare it to the preceding equation to see that it is the equation of an ellipse with:

   (b) Center where?

   (c) Horizontal and vertical semiaxes what?

   (d) Use techniques you've learned for sketching polar graphes to sketch the ellipse $r = 1/(1 + \frac{1}{2}\cos\theta)$.

2. (§H1) Find the value of θ in $0 < \theta < \pi$ at which the tangent line to the polar equation $r = 1/(1 + .75\cos\theta)$ is horizontal. What is the value of x at that point?

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3. (§H2) Consider the polar curves $r = 1 + \cos\theta$ and $r = 3\cos\theta$.

   (a) By hand, plot these curves on the same graph.

   (b) Find the area of the region that lies inside both $r = 1 + \cos\theta$ and $r = 3\cos\theta$ by setting up the correct integrals, with the correct limits, and then evaluating them.

4. (#34 on p. 652 of §9.1 and §9.7) For the points in space $A(-1, 5, 3)$ and $B(6, 2, -2)$, let S be the set of all points $P(x, y, z)$ such that the distance from P to A is twice the distance from P to B.

   (a) Find an equation for S from which you can show that S is a sphere of radius R and center at the point C. Determine R and the coordinates of C.

   (b) Follow the examples in §6.5.3 of ML and use MATLAB to plot the points A and B and the line segment joining them, and to plot the sphere S all on the same graph. Use the spherical coordinates θ and φ to parameterize S, but let θ range over $\alpha \leq \theta \leq \alpha + \frac{11}{6}\pi$, where you must figure out a value for α for which the wedge created in the sphere allows you to see the point B inside the sphere from MATLAB's default view. Set axis equal and label the points A and B either with the text command or by hand.

   A suggestion for how to proceed is to begin with $\alpha = 0$ and use the lines

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alpha = 0;
[theta,phi] = meshgrid(alpha:.2:alpha+11*pi/6,linspace(0,pi,20));
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   in your script. After you run the script, you will see where the opening is and then you can determine (even by trial and error) what value of α will work. Each change of value of α only requires a change in the line *alpha = 0.* Keep in mind that you can use negative values of α.