HOMEWORK 5, MATH 233
DUE MONDAY, FEBRUARY 11, 2002

NAME ____

Each problem is worth 6 points for a total of 30 points.

(1) (§10.2, #28 p.716)
   (a) Find the point $P(x, y, z)$ at which the curves $\mathbf{r}_1(t) = \langle t, 1-t, 3+t^2\rangle$ and $\mathbf{r}_2(s) = \langle 3-s, s-2, s^2\rangle$ intersect.
   (b) Find the angle of intersection correct to the nearest degree.

(2) (§10.3, like #39 p.724) Find equations of the osculating circles of the ellipse

$$x^2/9 + 4y^2 = 1$$

at the points $(3, 0)$ and $(0, 1/2)$. By hand, or with MATLAB, plot the ellipse and the two osculating circles on the same graph.

(3) (§10.3, like #37 p.724)
   (a) Find an equation of the osculating plane to the helix $\mathbf{r}(t) = \langle a\cos t, a\sin t, bt\rangle$ at $t = t_0$, where a and b are given positive constants.
   (b) By hand, or with MATLAB, plot a portion of the helix with the osculating plane for the case $a = 2$, $b = 1/2$ and $t_0 = \pi/2$.

(4) (§10.3, #50 p.725) Consider the problem of designing a railroad track to make a smooth transition between sections of straight track. Existing track along the negative x-axis is to be joined smoothly to a track along the line $y = 1$ for $x \geq 1$.
   Find a polynomial $P(x)$ of degree 5 such that the function F defined by

$$F(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ P(x) & \text{if } 0 < x < 1 \\ 1 & \text{if } x \geq 1 \end{cases}$$

is continuous and has continuous derivative and continuous curvature.

(5) (§10.4, like #31 p.734) Find the tangential and normal components of the acceleration vector for the motions whose position vector is $\mathbf{r}(t) = \cos 2t\, i + \cos 2t\, j + 2t\, k$.