HOMEWORK 6, MATH 233
DUE MONDAY, FEBRUARY 18, 2002

NAME _____

The first two problems are worth 8 points apiece and the last two are worth 7 points apiece for a total of 30 points.

(1) (§10.5, #32 p.742 and ML §7.4.3 p.128)
   (a) Find a parametric representation for the torus obtained by rotating about the z-axis the circle in the xz-plane with center $(b, 0, 0)$ and radius $a < b$. Write by hand these equations in the top margin of the graph you print out in the next part.
   (b) Use MATLAB and the equations you have obtained to plot the torus for the case $a = 1$ and $b = 3$. Set axis equal to get an undistorted figure.

(2) (§10.5 p.742) Find the parametric equations among problems 11 - 16 on page 741 of the helicoid in Figure I on page 741 and use these equations with MATLAB to plot two full turns, with the ramp extending out from the z-axis for a distance of 2 units from the z-axis. In the top margin of the printed graph write the parametric equations you used.

(3) (ML §7.4, #6 p.129)
   (a) Use the hyperbolic trigonometric functions to find a vector valued function $\mathbf{r}(u, v)$ whose image is the hyperboloid of one sheet $x^2 + y^2 - z^2 = 1$. Write your expression for $\mathbf{r}(u, v)$ in the upper margin of the graph you print out in the next part. Hint: Recall how to use spherical coordinates θ, φ to parameterize the sphere $x^2 + y^2 + z^2$ and then think about how to adjust this parameterization for the hyperboloid using $\cosh t$ and $\sinh t$ which satisfy $\cosh^2 t - \sinh^2 = 1$ for all t.
   (b) Use MATLAB with the parametric equations obtained in the first part to plot the part of the hyperboloid for which $-2 \leq z \leq 2$.

(Continued on back)

(4) (§10.4, Kepler's First Law, pp. 731 - 732) Suppose that the position of a particle at time t is $\mathbf{r}(t)$ and satisfies

$$\mathbf{r}'' = -\frac{5}{r^2}\mathbf{u}, \qquad \text{where } \mathbf{u} = \frac{1}{r}\mathbf{r} \text{ and } r = |\mathbf{r}|$$

where primes denote derivatives with respect to t. Let $\mathbf{h} = \mathbf{r} \times \mathbf{r}'$.

(a) Find $\mathbf{h}'$.

(b) Find $\mathbf{h} - r^2\mathbf{u} \times \mathbf{u}'$.

(c) By hand, sketch a diagram showing the vectors $\mathbf{u}$, $\mathbf{u}'$ and $\mathbf{u} \times \mathbf{u}'$, all based at the same point. Remember that $|\mathbf{u}| = 1$, so that $\mathbf{u}$ and $\mathbf{u}'$ are orthogonal. Arguing from your diagram, without further calculations, find $\mathbf{u} \times (\mathbf{u} \times \mathbf{u}') + \mathbf{u}'$.

(d) Find $(\mathbf{r}' \times \mathbf{h} - 5\mathbf{u})'$.

(e) Suppose that $\mathbf{h}(0) = \mathbf{k}$ and that $\mathbf{r}'(0) \times \mathbf{h}(0) - 5\mathbf{u} = 4\mathbf{i}$, where $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are the standard basis vectors. Find $\mathbf{r}(t) \cdot \mathbf{k}$, for all t.

(f) Define the function $\theta = \theta(t)$ by $\mathbf{u}(t) \cdot \mathbf{i} = \cos\theta$ and $\mathbf{u}(t) \cdot \mathbf{j} = \sin\theta$. Calculate both sides of the equation $(\mathbf{r} \times \mathbf{r}') \cdot \mathbf{h} = \mathbf{r} \cdot (\mathbf{r}' \times \mathbf{h})$ and solve for $r = f(\theta)$.

(g) By hand or with MATLAB, plot the trajectory of $\mathbf{r}(t)$ by plotting the polar curve $r = f(\theta)$.