HOMEWORK 7, MATH 233
DUE MONDAY, FEBRUARY 25, 2002

There are 30 points, allocated as shown in brackets.

(1) (See #2 p. 134 of ML and #4 p. 765, §11.2) Consider the function

$$f(x,y) = \frac{4xy}{x^2+3y^2}$$

(a) [2] Plot the graph of this function over the domain $-1 \le x \le 1$, $-1 \le y \le 1$. Experiment with the view to see which best shows the features of the graph at $(0,0)$. Use eps to prevent a gap at the origin. Use a fine grid, say at increments of .02, to get adequate detail.

(b) [2] Make a separate contour map of this function over the given domain. Use a finer grid, say with increments of .01. Specify the levels to run from -1.1 to 1.1 by increments of .1. Use the clabel command to label the contours at levels -1, 0 and 1.

(c) Write the following work in the space above and below your printed graph, continuing onto the bottom of the printed contour map, if necessary.

(i) [1] Show that restricted to any line through the origin, this function is constant. For example, do this by using polar coordinates to see that this function depends only on θ. How does this agree with the contour map?

(ii) [1] From this information, what can you conclude about $\lim_{(x,y)\to(0,0)} f(x,y)$?

(2) [6] (#36 p.777 of §11.3) Consider the function $f(x,y) = \sin(2x+3y)$. Use MATLAB to plot the graph of $z = f(-6,y)$. (That's a curve in the yz-plane. Label the axes correctly). On the print out use a straight edge to draw the tangent line to the graph at $y = 4$. In the top and bottom margins find $f_y(-6,4)$ and explain what this has to do with the slope of the line you have drawn on the graph.

(3) (#8 p.788 §11.4 and ML §8.3.1 pp. 136-137). Consider the function

$$f(x,y) = \frac{\sqrt{1+4x^2+4y^2}}{1+x^4+y^4}$$

(a) [3] Find the linearization, $L(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$.

(b) [3] Use MATLAB to plot on the same figure the graph of $z = f(x,y)$ over $[0,2]\times[0,2]$ and the graph of $z = L(x,y)$ over the same domain.

(Continued on back)

(4) (#2 p.138 §8.3.5 of ML). Consider the parametric surface traced out by the vector function

$$\mathbf{r}(s,t) = \langle 2\cos s \sin t, \sin s \sin t, 3\cos t \rangle$$

(a) [3] Find an equation of the tangent plane to this surface at the point $\mathbf{r}(0, \pi/3)$, or find parametric equations for the tangent plane by finding a vector function which traces out this tangent plane.

(b) [3] Use MATLAB to plot in the same figure the graph of $\mathbf{r}(s,t)$ over $0 \leq s \leq 2\pi$, $0 \leq t \leq \pi$, and an appropriate portion of the tangent plane at $\mathbf{r}(0, \pi/3)$.

(5) (See #28 on page 797 of §11.5) Wheat production in a given year, W, depends on the average temperature T and the annual rainfall R. Scientists estimate that the average temperature is rising at a rate of 0.15° C/year and rainfall is decreasing at a rate of 0.1 cm/year. They also estimate that, at current production levels, $\frac{\partial W}{\partial T} = -2$ and $\frac{\partial W}{\partial R} = 8$.

(a) [2] What is the significance of the signs of these partial derivatives?

(b) [4] Estimate the current rate of change of wheat production, $\frac{dW}{dt}$.