HOMEWORK 8, MATH 233
DUE MONDAY, MARCH 18, 2002

There are five problems, each worth 6 points. Points are allocated as shown in brackets for a total of 30 points.

(1) (Like #34, page 810 of §11.6) Consider $f(x, y) = x^2 - y^2$

- (a) [3] Find an equation of the tangent line to the level curve $f(x, y) = 5$ at the point $(3, 2)$.
- (b) [3] By hand, sketch the level curve, the tangent line and the gradient vector $\nabla f(3, 2)$. Label the y-intercept of the tangent line.

(2) (See #28 on page 809 of §11.6) The temperature at a point (x, y, z) is given by $T(x, y, z) = 200e^{-x^2-3y^2-9z^2}$ °C, where x, y and z are measured in meters.

- (a) [3] Find the rate of change of temperature (in °C/m) at the point $P(2, -1, 2)$ in the direction from P toward the point $Q(3, -3, 3)$.
- (b) [2] In which direction does the temperature increase fastest at P? Answer with a unit vector.
- (c) [1] Find the maximum rate of increase of T at P.

(3) (#10 page 819, §11.7)

- (a) [2] Find the local maximum and minimum values and saddle point(s) of the function $f(x, y) = 2x^3 + xy^2 + 5x^2 + y^2$
- (b) [2] Use MATLAB to plot the graph of this function over the domain $-2 \leq x \leq 1$, $-3 \leq y \leq 3$. Choose a good view. By hand, mark the critical points on the graph and label them. From this graph estimate the min and max of f on this domain. Use these values in the definition of the vector of levels you use in part c) below.
- (c) [2] Use MATLAB to make a contour plot of this function over the same domain used in part a). Use increments of .2 for the contours and have Matlab label those at level 0, and those above and below that level by increments of 5. By hand, mark the critical points on the contour map and label each as local maximum, local minimum or saddle point.

(Continued on next page)

(4) (Like #28 on page 819 of §11.7) Find the absolute maximum and minimum of $f(x,y) = 2x^2 + x + y^2 - 2$ on $D = \{(x,y) : x^2 + y^2 \leq 4\}$ as follows:

(a) [2] Find the critical points of f in the interior of D. Use the second derivative test to classify them as local maxima, local minima or saddle points. Evaluate f at each critical point.

(b) [2] Parameterize the boundary of D so that the restriction of f to the boundary is a function of the parameter. Find the critical points of this resulting function of one variable. Evaluate f at each critical point.

(c) [2] Explain how to use your information from parts (a) and (b) to find the absolute maximum and minimum values and points of f on D. Mark and label these points on a hand sketch of D.

(5) (See #4, page 828 of §11.8)

(a) [3] Use Lagrange multipliers to find the maximum and minimum values of the function $f(x,y) = 2x + y$ subject to the constraint $x^2 + 4y^2 = 1$.

(b) [2] Use MATLAB to plot the constraint curve on a contour map of f. On the contour map include the levels $f(x,y) = M$ and $f(x,y) = m$, where M is the maximum of f and m is the minimum value found in part (a). Use the clabel command to label the levels. Set axis equal.

(c) [1] By hand, mark the points on the constraint where f achieves its maximum and minimum, respectively. Draw $\frac{1}{4}\nabla f$ and $\frac{1}{4}\nabla(x^2 + 4y^2)$ evaluated at these points, respectively. (The factor of $\frac{1}{4}$ is there so that the vector will fit in the picture).