# HOMEWORK 9, MATH 233
# DUE MONDAY, MARCH 25, 2002

There are five problems, each worth 6 points. Points are allocated as shown in brackets for a total of 30 points.

(1) (Cf. Example 5, p.827, §11.8) The plane $x + y + z = 24$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse. Find the highest and lowest points on the ellipse. Here height means the distance above the $xy$-plane (or below, which would be a negative height). Show the following steps.

   (a) [2] For what function $f(x, y, z)$ do you want to find the extreme points? What are the constraints?

   (b) [4] Carry out the method of Lagrange multipliers to solve the problem you have set up in part (a).

(2) [6] (See #34 on page 912, review of §12.4) Use polar coordinates to find the volume of the solid under the half-cone $z = \sqrt{x^2 + y^2}$ and above the paraboloid $z = x^2 + y^2$. Sketch the solid by hand.

(3) (Like #21, p.877, §12.5). Let

$$f(x, y) = \begin{cases} ce^{-(.3x+.4y)} & \text{if } x \geq 0,\ y \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

where $c$ is some positive constant.

   (a) [2] For what value of $c$ is $f(x, y)$ a joint density function for random variables $X$ and $Y$?

   (b) [2] Find the probability $P(X \leq 2, Y \leq 4)$.

   (c) [2] Find the expected value of $X$.

(4) [6](See #18 p.861, §12.3) Consider the solid region lying below the graph of $z = 3x^2 + y^2$ and above the domain $D$ bounded by the curves $y = x$ and $x = y^2 - y$. This problem asks you to approximate the volume of this region by calculating a Riemann sum. The next problem asks you to find the exact value of the volume by calculating an iterated integral. A MATLAB script for generating a 3-d graphic of this solid region is available at

   `http://www.math.wustl.edu/~gary/Math233/Spring02/H9Solid.m`

Use cut and paste to copy this file and run it. This graphic gives a good picture of the solid whose volume you are trying to calculate. A study of the script might be helpful in carrying out the problem below.

Convince yourself that the rectangular region $R = [-.25, 2] \times [0, 2]$ contains the domain $D$. Take a regular partition of $R$ with $\Delta x = 2.25/n$ and $\Delta y = 2/n$, where $n = 100$. Use the midpoints $(x_{ij}^*, y_{ij}^*)$ as sample points, $i, j = 1, \ldots, 100$. Use MATLAB to calculate the Riemann sum

$$RS = \sum_{i=1}^{100} \sum_{j=1}^{100} F(x_{ij}^*, y_{ij}^*)\, \Delta A$$

where

$$F(x, y) = \begin{cases} 3x^2 + y^2 & \text{if } (x, y) \text{ is in } D, \\ 0 & \text{otherwise.} \end{cases}$$

and $\Delta A = \Delta x\, \Delta y$. Graph the domain $D$ in the $xy$-plane (by graphing its boundary curves) and report the value of RS in the title. Print this 2-d graph and do the next problem on its bottom margin.

*Hints for writing the script:* Use *meshgrid* to generate the coordinates of the midpoints. Here is a start, where $dx = \Delta x$ is assumed already defined in the script. You fill in the space where there are three dots.

```
[x,y] = meshgrid(-1/4+dx/2:dx:2-dx/2,...);
z = 3*x.^2+y.^2;
```

To make $z = 3x^2 + y^2$ be zero outside of $D$, use relational operators:

```
Z = ((x-y.^2+y)>=0).*((x-y)<=0).*z;
```

Notice that the output of, for example, the relational operator $(x - y) <= 0$ is a matrix the same size as x and y in which an entry is 0 when this inequality is false for that entry of x and y, and is 1 when it is true. The two inequalities in the relational operators are exactly the inequalities defining $D$.

(5) [6] On the bottom margin of your printed graph from the preceding problem, set up and evaluate an iterated integral for the volume of the solid region of that problem. Of the calculation, you need show only the results after the first partial integration and then your answer.