

HOMEWORK 12, MATH 233
THE LAST ONE
DUE MONDAY, APRIL 15, 2002

Points are allocated as shown in brackets for a total of 30 points.

- (1) [5] (#35 and #36 p.935 §13.2) A 160-lb man carries a 25-lb can of paint up a helical staircase that encircles a silo with a radius of 20 ft. The silo is 90 ft high and the man makes exactly three complete revolutions in climbing to the top. If there is a hole in the can of paint and 9 lb of paint leak steadily out of the can during the man's ascent, how much work is done by the man against gravity in climbing to the top?

- (2) (#16 p.944 §13.4) Consider the vector field

$$\mathbf{F}(x, y, z) = (2xz + y^2) \mathbf{i} + 2xy \mathbf{j} + (x^2 + 3z^2) \mathbf{k}$$

on \mathbf{R}^3 .

- (a) [4] Verify that \mathbf{F} is conservative and find a function $f(x, y, z)$ on \mathbf{R}^3 such that $\nabla f = \mathbf{F}$.
- (b) [1] Use part (a) to find $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve given by the parametric equations $x = t^2$, $y = t + 1$, $z = 2t - 1$, over $0 \leq t \leq 1$.

- (3) [5] (#10 p.951 §13.4) Use Green's Theorem to evaluate the line integral

$$\int_C (y^2 - \arctan x) dx + (3x + \sin y) dy$$

where C is the boundary of the region enclosed by the parabola $y = x^2$ and the line $y = 4$.

- (4) [5] (#20 p.951 §13.4) Use one of Green's formulas to find the area of the region bounded by the curve with vector equation $\mathbf{r}(t) = \cos t \mathbf{i} + \sin^3 t \mathbf{j}$, over $0 \leq t \leq 2\pi$.

Continued on next page

- (5) (See #33 page 945 of §13.3 and §10.1.1 of ML) Consider $\mathbf{F}(x, y) = -\frac{y}{x^2+y^2} \mathbf{i} + \frac{x}{x^2+y^2} \mathbf{j}$ on the domain D , which consists of the points in the square $[-\sqrt{2}, \sqrt{2}] \times [-\sqrt{2}, \sqrt{2}]$ with the disk $\{(x, y) : x^2 + y^2 < \frac{1}{4}\}$ removed.
- (a) [2] Use the quiver command to plot \mathbf{F} on D . To do this, use meshgrid to define $[x,y]$ on the square, and then use the command lines
`index = find((x.^2 + y.^2)<1/4); x(index) = NaN*x(index)`
to remove the central disk. Set axis equal. On the same graph plot the circle C of radius 1 with center at the origin. Do the following parts on the top and bottom margins of this graphic.
- (b) [1] Without making any calculations, explain why you think the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ is, or is not, zero, where C is traversed once in the counterclockwise direction.
- (c) [2] Calculate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$. According to your calculation and Theorem 3 on page 938, is $\int_{\tilde{C}} \mathbf{F} \cdot d\mathbf{r}$ independent of path \tilde{C} in D ?
- (d) [1] Calculate $\frac{\partial}{\partial y}(-\frac{y}{x^2+y^2})$ and $\frac{\partial}{\partial x}(\frac{x}{x^2+y^2})$. From the results of this calculation can you conclude that \mathbf{F} is conservative on D ? Explain your answer.
- (6) [4] (#2 p.989 Focus on Problem Solving) This problem uses only material you have learned about calculus in this course and its prerequisites, but it is unlike the examples and exercises you have seen. The solution to this problem is not complicated or long, but it requires you to think outside of the model problems you have seen so far and it requires some careful reasoning. Be sure to explain each step in your solution. Please give this problem a try. Work with others on it. At first you might feel clueless, but if you persist to a solution, you will feel great satisfaction in your accomplishment.

Find the simple closed curve C for which the value of the line integral

$$\int_C (y^3 - y) dx - 2x^3 dy$$

is a maximum.