

Exam 1, Math 266
February 23, 2000

All numbers are written in base 10 numerals except where stated otherwise. Each problem is worth 2 points. If a problem has two parts, each part is worth 1 point. There are 30 points.

1. Give a story problem whose solution illustrates
 - (a) the definition of $3 + 5$ (union of disjoint sets).
 - (b) the number line model of $4 + 2$.
2. (a) Write the definition of $7 - 4$ (opposite of addition).
(b) Describe the set model for $5 - 3$.
3. In order to explain why $7 - (2 + 3) = (7 - 2) - 3$, one must show that $((7 - 2) - 3) + (2 + 3) = 7$, by definition of $7 - (2 + 3)$. Use the commutative and associative properties of addition together with the definition of subtraction to give an explanation of why $((7 - 2) - 3) + (2 + 3) = 7$.
4. Give a story problem whose solution illustrates $21 \div 3$ by
 - (a) the measurement model of division.
 - (b) the partitive model of division.
5. Use the array model of multiplication to illustrate the meaning of the division algorithm for $14 \div 4$, that is, to illustrate $14 = q \cdot 4 + r$, where q is the quotient and r is the remainder.
6. (a) Find the base 5 numeral of one-hundred-sixty-four.
(b) Give a detailed explanation of your method in terms of composing 5's, then composing 5's of 5's, etc. Your explanation should include the role played by the division algorithm in this process. For example, why is the digit in the one's place equal to the remainder in $164 \div 5$, and the digit in the five's place is the remainder of what division, and so on?
7. Explain how to use the scratch addition algorithm to compute the sum $88 + 65 + 39$. In your explanation mention what are the basic addition facts needed and what each scratch means in terms of composition of 10 units in a place into 1 unit in the next place to the left.

8. Carry out, with margin notes, the subtraction algorithm for $234 - 186$.
9. Compute $47 \cdot 563$ in columns with margin notes.
10. Compute $582 \div 6$ with margin notes.
11. Compute $8112 \div 19$ with margin notes.
12. (a) In base 5 numeration compute $231 \div 4$, where these are base 5 numerals. Include margin notes.
(b) In base 2 numeration compute $110 \cdot 101$ in columns with margin notes, where these are base 2 numerals.
13. Write time as hours:minutes:seconds. If the exam ends at 5:36:38 and it started at 3:56:58 (both times in the same afternoon), how long did it last? Explain what computation must be done and write it out, with margin notes, showing how it resembles one of the arithmetic algorithms for numerals in base 60.
14. Explain how to compute $9746 \div 35$ by factoring $35 = 5 \cdot 7$ and then dividing first by 5, then by 7, keeping track of the remainders and what they mean. Indicate clearly the quotient q and remainder r . Verify that your answer is correct by computing $q \cdot 35 + r$ and seeing that it equals the dividend.
15. For whole numbers a and b , we defined $a < b$ to mean that if $a = n(A)$ and if $b = n(B)$, for some sets A and B , then A is in one-to-one correspondence with a proper subset of B . Give an example of a situation in which one uses this idea to determine that the number of objects in some set is smaller than the number of objects in another set, without actually counting the objects. In your example, explain what is the set A , what is the set B and how the one-to-one correspondence between A and a proper subset of B is achieved. You may use examples discussed in class, in the text, in the lecture notes or that you make up yourself.