

Calculus III

Math 233 — Spring 2007

In-term exam 02/07

This problem set contains sixteen problems numbered 1 through 16. Problems 1 – 15 are multiple choice problems, which each count 5% of your total score. Problem 16 will be hand-graded and counts 25% of your total score.

Problem 1

What is the distance from the point $P(3, -3, 4)$ to the x -axis?

- A) 0 B) 3 C) 4 D) 5 E) $\sqrt{34}$ F) None of the above

Problem 2

Which of the following equations describes a plane parallel to $2x - y + 4z + 4 = 0$?

- A) $x - y + z + 2 = 0$ B) $y = 2(x + z)$ C) $2x^2 - y^2 + 4z^2 + 4 = 0$
D) $-x + \frac{1}{2}y - 2z = 0$ E) $2x + y + 4z = 4$ F) None of the above

Problem 3

Find a vector parallel to the line described by

$$\frac{x - 2}{7} = \frac{y}{3} = \frac{2 - z}{5}.$$

- A) $\langle 2, 0, 2 \rangle$ B) $\langle 7, 3, 5 \rangle$ C) $\langle \frac{2}{7}, \frac{1}{3}, \frac{-2}{5} \rangle$ D) $\langle -14, -6, 10 \rangle$ E) $\langle 1, 0, -1 \rangle$
F) None of the above

Problem 4

Convert the point given by the cylindrical coordinates $(4, \frac{\pi}{3}, -1)$ to rectangular (Cartesian) coordinates.

- A) $(1, \sqrt{3}, 1)$ B) $(\sqrt{2}, \sqrt{2}, -1)$ C) $(\sqrt{3}, 1, 1)$ D) $(2, 2\sqrt{3}, -1)$
E) $(2\sqrt{3}, 2, -1)$ F) None of the above

Problem 5

What is the scalar projection of $\langle -6, 1, 7 \rangle$ onto $\vec{i} + 4\vec{j} - 2\vec{k}$?

- A) $\frac{16}{\sqrt{21}}$ B) $\frac{16}{\sqrt{86}}$ C) $\frac{-16}{\sqrt{21}}$ D) $\frac{-16}{\sqrt{86}}$ E) $\sqrt{906}$
F) None of the above

Problem 6

The lines $\langle 1+3t, -1, 4-3t \rangle$ and $\langle 1+t, 1-t, 2 \rangle$ intersect in the point $(3, -1, 2)$. What is the angle between the two lines?

- A) 0 B) $\frac{\pi}{6}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{3}$ E) $\frac{\pi}{2}$ F) None of the above

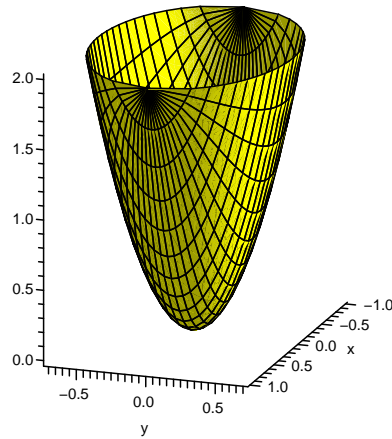
Problem 7

Write the equation $z = x^2 + y^2$ in spherical coordinates.

- A) $\rho^2 = \cos \phi$ B) $\rho \sin \phi \tan \phi = 0$ C) $\cos^2 \phi = \sin^2 \phi$
D) $\cos \phi = \rho \sin^2 \phi \cos 2\theta$ E) $\rho \sin^2 \phi = \cos \phi$ F) None of the above

Problem 8

What is the equation of the following surface?



- A) $y = \frac{1}{2}x^2 + \frac{1}{4}z^2$ B) $z = x^2 - \frac{1}{2}y^2$ C) $y = z^2 - \frac{1}{2}x^2$ D) $y^2 = x^2 + 2z^2$
E) $z = 2x^2 + 4y^2$ F) None of the above

Problem 9

Which of the following expressions is meaningful?

- A) $\vec{a} \cdot \vec{b} + \vec{c}$ B) $|\vec{a}| \times (\vec{b} \cdot \vec{c})$ C) $(\vec{a} \cdot \vec{b}) \times \vec{c}$ D) $(\vec{a} + \vec{b}) \cdot (\vec{a} \times \vec{c})$ E) $\vec{a} \cdot \vec{b} \cdot \vec{c}$
F) None of the above

Problem 10

Which of the following vectors is parallel to the plane described by

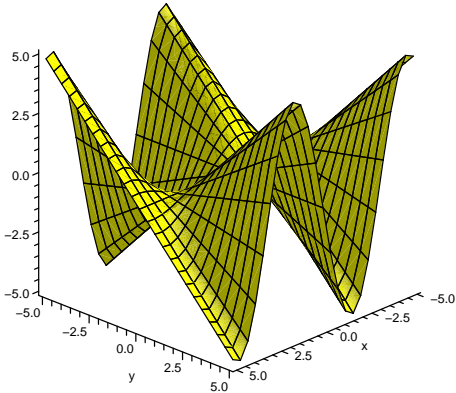
$$3(x - 2) + (y - 7) - 4(z + 1) = 0?$$

- A) $\langle 1, 1, 1 \rangle$ B) $\langle 1, 1, 9 \rangle$ C) $\langle 2, 7, -1 \rangle$ D) $\langle 3, 1, -4 \rangle$ E) $\langle -6, -7, 4 \rangle$
F) None of the above

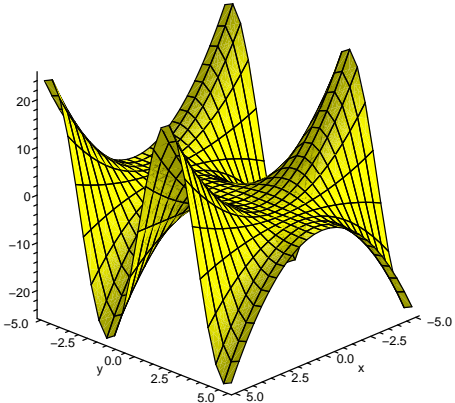
Problem 11

Sketch the graph of the function $f(x, y) = y^2 \cos x$.

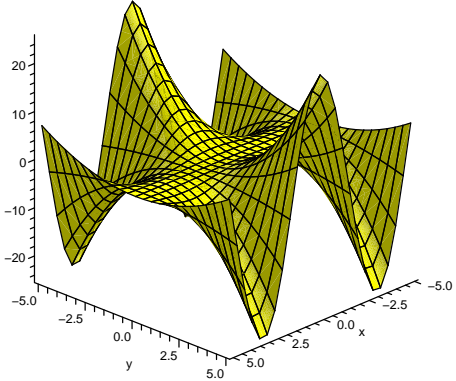
A)



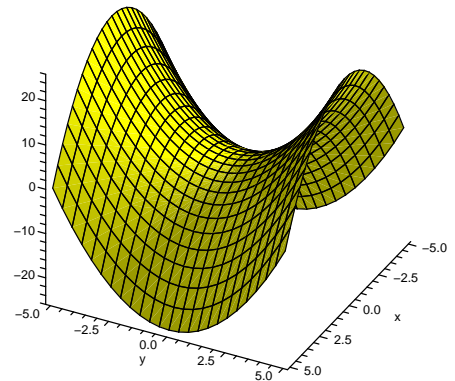
B)



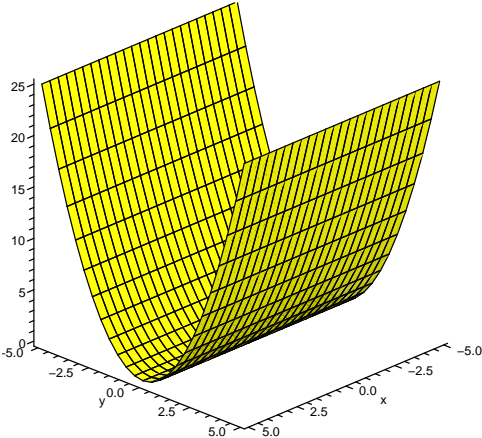
C)



D)



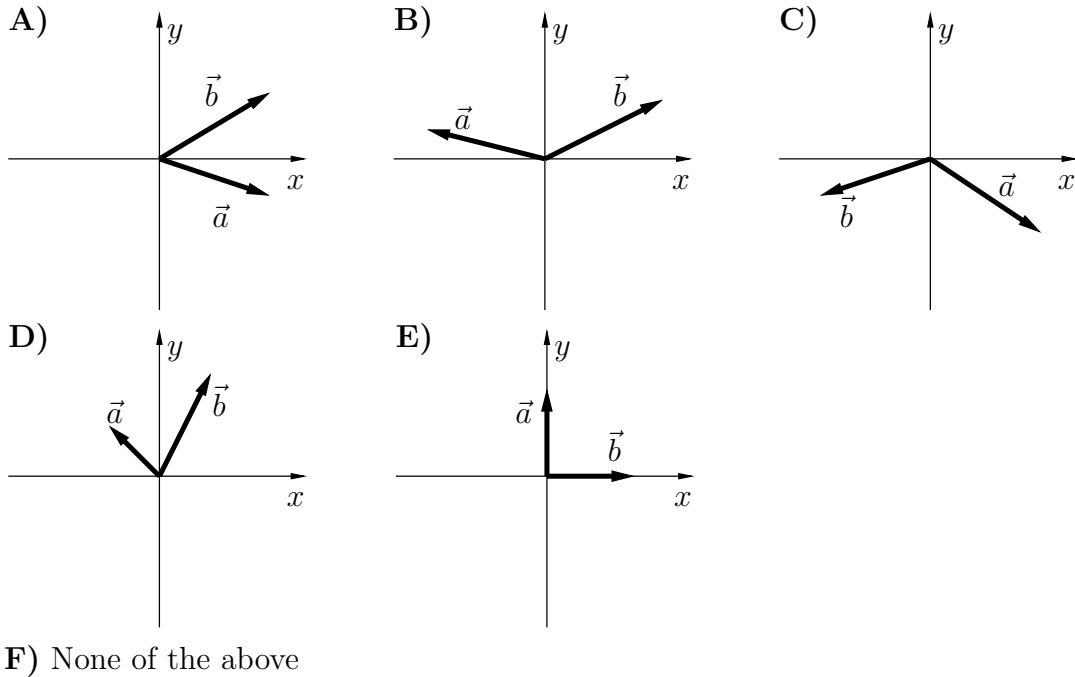
E)



F) None of the above

Problem 12

Which of the following figures depicts two vectors \vec{a} and \vec{b} in the xy -plane such that $\vec{a} \cdot \vec{b} < 0$, $\vec{a} \times \vec{b}$ is parallel to $-\vec{k}$, the x -component of $\vec{a} + \vec{b}$ is 0, and the y -component of $2\vec{a} - \vec{b}$ is 0?



Problem 13

Which of the following is the equation of a line that lies in both the planes

$$x - 3y + 2z = 2 \quad \text{and} \quad x + y = 0?$$

- A) $\langle \frac{1}{2} + t, -\frac{1}{2} + t, 0 \rangle$ B) $\langle 1 - t, 3 + t, 2 + 2t \rangle$ C) $\langle t, -t, 1 - 2t \rangle$
 D) $\langle 1 + t, -1 - 3t, -1 + 2t \rangle$ E) $\langle 1 + 2t, 1 - 6t, 4t \rangle$ F) None of the above

Problem 14

Which of the following equations describes a sphere of radius 3?

- A) $3x^2 + 3y^2 + 3z^2 = 0$ B) $x^2 - y^2 + 9 = z^2$ C) $x^2 + y^2 + z^2 - 2x + 4z = 4$
 D) $x^2 + y^2 + z^2 - y + 2z = 9$ E) $z = x^2 + y^2 - 9$ F) None of the above

Problem 15

What is the angle between the planes $x = y$ and $z = \frac{1}{2}\sqrt{2}(x - y)$?

- A) 0 B) $\frac{\pi}{6}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{3}$ E) $\frac{\pi}{2}$ F) None of the above

The following problem will be hand-graded. To earn full credit you need to justify your answers.

Problem 16

- a) Let \vec{a} and \vec{b} be two vectors. Show that the area of the parallelogram spanned by \vec{a} and \vec{b} is the same as the area of the parallelogram spanned by \vec{a} and $\vec{a} + \vec{b}$.
- b) Let A_1 denote the area of the parallelogram spanned by \vec{a} and \vec{b} , and let A_2 denote the area of the parallelogram spanned by $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$. Find the ratio $\frac{A_1}{A_2}$.