

Calculus III

Math 233 — Spring 2007

In-term exam March 7th.

This exam contains sixteen problems numbered 1 through 16. Problems 1 – 15 are multiple choice problems, which each count 5% of your total score. Problem 16 will be hand-graded and counts 25% of your total score.

Problem 1

Find

$$\lim_{t \rightarrow 0} \frac{\sin t}{t} \vec{i} + t \vec{j} + (t - 1)^4 \vec{k}.$$

- A) $\vec{0}$ B) \vec{i} C) \vec{j} D) \vec{k} E) $\vec{i} + \vec{j}$ F) $\vec{i} + \vec{k}$ G) $\vec{j} + \vec{k}$
H) $\vec{i} + \vec{j} + \vec{k}$

Problem 2

What is the area of the triangle with vertices $(1, 2, 3)$, $(3, 1, 4)$, and $(7, 1, 8)$?

- A) 2 B) $2\sqrt{3}$ C) 4 D) 6 E) $4\sqrt{3}$ F) 8 G) 12
H) $8\sqrt{3}$

Problem 3

If $z = (3x - 7y)^{2007}$, where $x = 2 \cos t$ and $y = (t + 1)^2$, then $\frac{dz}{dt} = 2007 \cdot a$ at $t = 0$. Find a .

- A) -20 B) -14 C) -7 D) 0 E) 7 F) 8 G) 14
H) 4^{2006}

Problem 4

The surface described by $\vec{r}(u, v) = \cos u \vec{i} + \sin u \cos v \vec{j} + \sin u \sin v \vec{k}$ where $0 \leq u \leq \pi$ and $0 \leq v \leq 2\pi$ is a

- A) Cone B) Cylinder C) Elliptic hyperboloid D) Horizontal plane
E) Hyperbolic hyperboloid F) Hyperboloid of one sheet
G) Non-horizontal plane H) Sphere

Problem 5

What is the length of the arc described by $\vec{r}(t) = 3t^2 \vec{i} + 2t^3 \vec{j} + \vec{k}$, $0 \leq t \leq 1$?

- A) 1 B) $\sqrt{6} - 1$ C) 2 D) $\sqrt{6}$ E) $2(2\sqrt{2} - 1)$ F) $4\sqrt{2}$
G) 6 H) $6\sqrt{2}$

Problem 6

Let $f(x, y) = x^2 \cos \pi y - \ln 2xy$. Find $f_x(2, 3) + f_y(2, 3)$.

- A) $-4\pi - \frac{1}{6}$ B) $-\frac{29}{6}$ C) $-\frac{25}{6}$ D) $-\frac{9}{2}$ E) $-\frac{1}{3}$ F) 0 G) $\frac{19}{6}$
H) $4\pi - \frac{5}{6}$

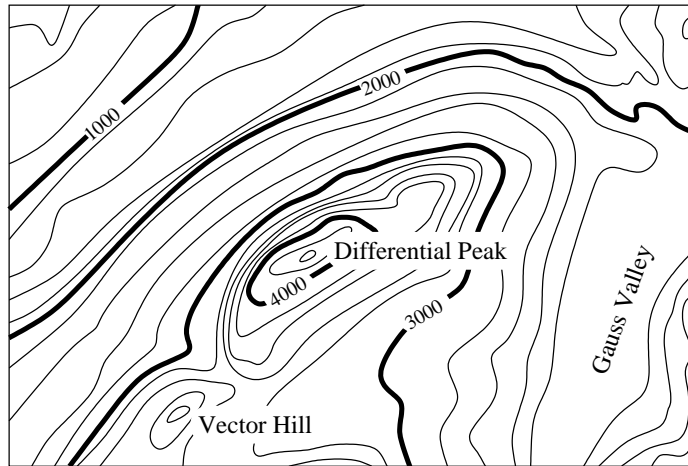
Problem 7

A cube has four diagonals. Find the acute angle between any two of them.

- A) 0° B) $\arccos \frac{1}{\sqrt{2}} = 45^\circ$ C) $\arccos \frac{1}{\sqrt{3}} \approx 54.7^\circ$ D) $\arccos \frac{1}{2} = 60^\circ$
E) $\arccos \frac{1}{2\sqrt{2}} \approx 69.3^\circ$ F) $\arccos \frac{1}{3} \approx 70.5^\circ$ G) $\arccos \frac{1}{2\sqrt{3}} \approx 73.2^\circ$
H) $\arccos \frac{1}{3\sqrt{3}} \approx 78.9^\circ$

Problem 8

Use the given topographic map of Calculus Mountain to estimate the height difference between the top of Differential Peak and the top of Vector Hill.



- A) 0 B) 200 C) 300 D) 400 E) 600 F) 1000 G) 1600
H) 1800

Problem 9

Find the partial derivative $\frac{\partial z}{\partial x}$ at the point $(0, 9, 10)$ where $z = f(x, y)$ is defined implicitly by

$$2x^3 + y^3 + z^3 - xyz = 1729.$$

- A) $-\frac{1}{2}$ B) $-\frac{3}{10}$ C) $-\frac{21}{100}$ D) 0 E) $\frac{21}{100}$ F) $\frac{3}{10}$ G) $\frac{1}{2}$
H) 1729

Problem 10

Find the curvature at $t = 0$ of the curve described by

$$x = 2 - \cos t, \quad y = \sqrt{2} \sin t, \quad z = \cos t.$$

- A) 0 B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) $\frac{1}{\sqrt{2}}$ E) 1 F) $\sqrt{2}$ G) 2 H) 3

Problem 11

Let $\vec{u} = \frac{1}{\sqrt{2}}\langle 1, -1 \rangle$. Find the directional derivative $D_{\vec{u}}f(1, -1)$ when

$$f(x, y) = (x + 3y)^2.$$

- A) $-4\sqrt{2}$ B) $-2\sqrt{2}$ C) $-\sqrt{2}$ D) 0 E) $\sqrt{2}$ F) $2\sqrt{2}$
G) $4\sqrt{2}$ H) 8

Problem 12

For which of the points $(a, b) = (0, 0)$, $(a, b) = (1, -1)$, and $(a, b) = (1, 1)$ does the limit

$$\lim_{(x,y) \rightarrow (a,b)} \frac{[x^2(x-1) + y^2(x-1)](y-1)}{[(x-1)^2 + (y-1)^2](x^2 + y^2)}$$

exist?

- A) None of them B) Only $(0, 0)$ C) Only $(1, -1)$ D) Only $(1, 1)$
E) $(0, 0)$ and $(1, -1)$ F) $(0, 0)$ and $(1, 1)$ G) $(1, -1)$ and $(1, 1)$
H) $(0, 0)$, $(1, -1)$, and $(1, 1)$

Problem 13

Let $z = 4 + x^3 + y^3 - 3xy$. Which of the following statements are true?

- I) $(0, 0)$ is a saddle point,
II) $(1, 1)$ is a local maximum,
III) $(2, 4)$ is a local minimum.
A) None of them B) Only I C) Only II D) Only III E) I and II
F) I and III G) II and III H) I, II, and III

Problem 14

Mr. P. Article is subject to the acceleration $\vec{a}(t) = t\vec{i} + (1 - 3t^2)\vec{k}$. At time $t = 1$ his velocity is $\vec{v}(1) = \vec{j} + \vec{k}$ and his position is $\vec{r}(1) = 7\vec{j}$. Find $\vec{r}(2)$, his position at time $t = 2$.

- A) $\langle 0, 1, 1 \rangle$ B) $\langle 0, 7, 0 \rangle$ C) $\langle \frac{1}{6}, 0, \frac{1}{4} \rangle$ D) $\langle \frac{1}{6}, 8, \frac{5}{4} \rangle$ E) $\langle \frac{2}{3}, 8, -\frac{5}{4} \rangle$
F) $\langle \frac{2}{3}, 9, -\frac{5}{4} \rangle$ G) $\langle \frac{4}{3}, 0, -2 \rangle$ H) $\langle \frac{4}{3}, 9, 0 \rangle$

Problem 15

The plane $y = 1$ intersects the graph of $z = 3x^2y - xy^3 + 5y^2$ in a curve. The tangent line to this curve at the point $(1, 1, 7)$ passes through a point $(0, 1, c)$. What is c ?

- A) -12 B) -6 C) -2 D) 0 E) 2 F) 6 G) 12 H) 24

The following problem will be hand-graded. To earn full credit you need to justify your answers.

Problem 16

A surface S is given by $z = 4 - x^2 - y^2$.

- a) Find the equation of the tangent plane of S at the point $(1, 1, 2)$.
b) Let C be the curve where S intersects the surface defined by

$$x^2y = 1, \quad x > 0.$$

Think of C as a trail in a terrain described by S . How steep is the trail at the point lying directly above $(x, y) = (1, 1)$? Give the answer as an angle related to the horizontal plane.