Calculus III

Math 233 — Spring 2007

In-term exam April 11th.

This exam contains sixteen problems numbered 1 through 16. Problems 1 – 15 are multiple choice problems, which each count 5% of your total score. Problem 16 will be hand-graded and counts 25% of your total score.

Problem 1

Evaluate $\int_4^{11} \int_2^{33} y \sin(\pi x)\, \mathrm{d}x\, \mathrm{d}y$.

A) π **B)** 31π **C)** $\frac{14}{\pi}$ **D)** $\frac{105}{\pi}$ **E)** 0 **F)** $\frac{1}{\pi}$ **G)** 14 **H)** 7π

Problem 2

Use Lagrange multipliers to find the maximum value of the function

$$f(x, y, z) = x - y + z$$

on the sphere $(x-1)^2 + (y-2)^2 + (z-1)^2 = 4$.

A) $\frac{4}{3}\sqrt{3}$ **B)** $\frac{2}{3}\sqrt{3}$ **C)** $\frac{1}{4}\sqrt{3}$ **D)** $1 + \frac{2}{3}\sqrt{3}$ **E)** 1 **F)** $\frac{1}{6}\sqrt{3}$
G) $2\sqrt{3}$ **H)** $2 - \frac{2}{3}\sqrt{3}$

Problem 3

Find the surface area of the part of $z = 1 - x^2 - y^2$ that lies above the xy-plane.

A) $\frac{5\pi}{6}$ **B)** $\frac{\sqrt{3}\pi}{4}$ **C)** $\frac{2}{\pi}$ **D)** $1 + \frac{2}{3}\sqrt{\pi}$ **E)** $\frac{\pi}{\sqrt{5}}$ **F)** $\frac{(5\sqrt{5}-1)\pi}{6}$
G) $2\pi\sqrt{7}$ **H)** $\frac{2\sqrt{3}-1}{3}$

Problem 4

Find the volume of the solid bounded by $z = 1 - x^2 - y^2$ and the xy-plane.

A) $\frac{1}{2}$ **B)** $\frac{2}{3}$ **C)** 1 **D)** $\frac{4}{3}$ **E)** $\frac{\pi}{2}$ **F)** $\frac{2\pi}{3}$ **G)** π **H)** $\frac{4\pi}{3}$

Problem 5

Let D be the triangle with vertices $(0, 2)$, $(3, -1)$, and $(3, 2)$. Calculate

$$\iint_D xy \, \mathrm{d}A.$$

A) $\frac{63}{8}$ **B)** $\frac{37}{4}$ **C)** $\frac{57}{2}$ **D)** $\frac{51}{8}$ **E)** $\frac{41}{2}$ **F)** $\frac{33}{4}$ **G)** $\frac{47}{6}$ **H)** $\frac{11}{4}$

Problem 6

Find the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ of the transformation

$$x = 3u + v, \quad y = 4u - 2v.$$

A) -10 **B)** -5 **C)** -2 **D)** 0 **E)** 1 **F)** 3 **G)** 4 **H)** 6

Problem 7

Compute the volume of the pyramid with a rectangular base that is bounded by the five planes $3x + z = 12$, $-3x + z = 12$, $3y + z = 12$, $-3y + z = 12$ and $z = 0$.

Hint: Use symmetry to calculate the volume of the pyramid as four times the volume of a tetrahedron.

A) 64 **B)** 84 **C)** 128 **D)** 172 **E)** 216 **F)** 256 **G)** 342
H) 432

Problem 8

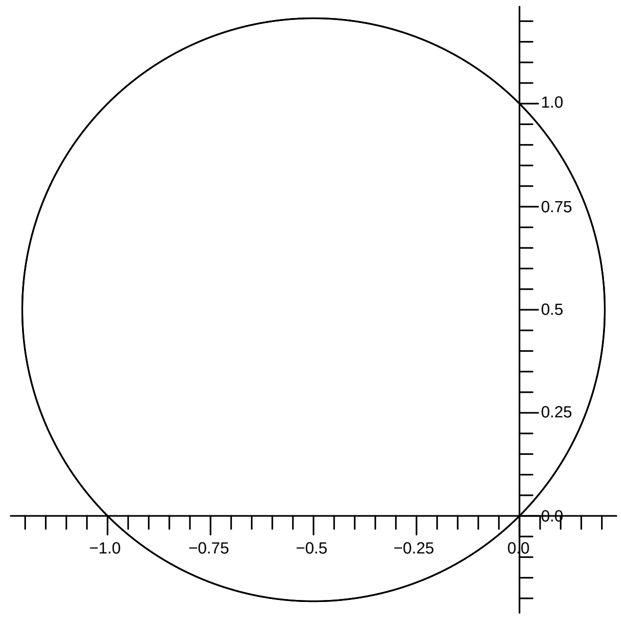
Evaluate $\iiint_E \frac{1}{x^2+y^2+z^2} \, \mathrm{d}V$, where E is the solid bounded by two spheres:

$$E = \{(x, y, z) \colon 1 \leq x^2 + y^2 + z^2 \leq 4\}.$$

A) $\frac{28\pi}{3}$ **B)** 4π **C)** $\frac{2}{3}\pi$ **D)** $\frac{4\pi}{3}$ **E)** $\frac{32\pi}{3}$ **F)** 3π **G)** 2π
H) π

Problem 9

Which of the following polar equations defines the circle below?



A) $r = 1-2\sin(\theta)$ **B)** $r = \cos(\theta)$ **C)** $r = \sin(\theta)-\cos(\theta)$ **D)** $r = \theta\cos(\theta)$
E) $r = \theta\sin(2\theta)$ **F)** $r = 2+\sin(2\theta)$ **G)** $r = \sin(\theta)-1$ **H)** $r = \theta$

Problem 10

Evaluate

$$\int_0^1 \int_{\sqrt{x}}^1 \cos\left(y^3\right) \, \mathrm{d}y \, \mathrm{d}x.$$

by changing the order of integration.

A) $\frac{1}{3}\left(\cos(1)-1\right)$ **B)** $\frac{1}{3}\cos(1)$ **C)** $\frac{1}{3}\left(\sin(1)-1\right)$ **D)** $\frac{1}{3}\sin(1)$
E) $\cos(1)-1$ **F)** $\cos(1)$ **G)** $\sin(1)-1$ **H)** $\sin(1)$

Problem 11

Find the tangent vector of the curve with parametric equations

$$x(t) = \int_0^t \sin\left(\tfrac{1}{2}\pi\theta^2\right) \mathrm{d}\theta, \qquad \text{and} \qquad y(t) = \int_0^t \cos\left(\tfrac{1}{2}\pi\theta^2\right) \mathrm{d}\theta$$

at the point corresponding to $t = 1$.

A) $\langle 1, 0\rangle$ **B)** $\langle \frac{1}{2}\sqrt{2}, \frac{1}{2}\sqrt{2}\rangle$ **C)** $\langle 0, 1\rangle$ **D)** $\langle -\frac{1}{2}\sqrt{2}, \frac{1}{2}\sqrt{2}\rangle$ **E)** $\langle -1, 0\rangle$
F) $\langle -\frac{1}{2}\sqrt{2}, -\frac{1}{2}\sqrt{2}\rangle$ **G)** $\langle 0, -1\rangle$ **H)** $\langle \frac{1}{2}\sqrt{2}, -\frac{1}{2}\sqrt{2}\rangle$

Problem 12

Describe the boundary of the solid region E that we integrate over in

$$\iiint_E f(x,y,z)\,\mathrm{d}V = \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{2-x^2-y^2} f(x,y,z)\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x.$$

A) A paraboloid and a plane **B)** A sphere **C)** A cone and a half-sphere
D) A paraboloid and a half-sphere **E)** A cone and a plane **F)** An ellipsoid
G) Two cones **H)** A cone and a paraboloid

Problem 13

Let D be the region bounded by the circle $x^2 + y^2 = 2x$. Evaluate

$$\iint_D (x^2 + y^2)y\,\mathrm{d}A.$$

A) 0 **B)** $\frac{1}{30}$ **C)** $\frac{1}{20}$ **D)** $\frac{1}{10}$ **E)** $\frac{\pi}{10}$ **F)** $\frac{\pi}{2}$ **G)** π **H)** 2π

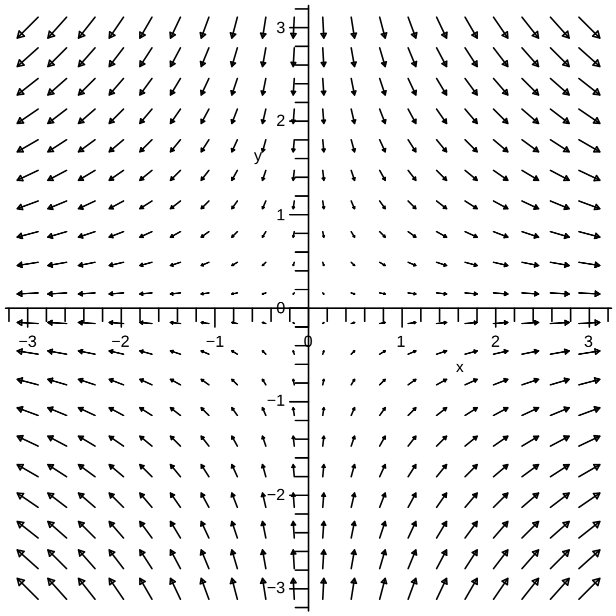
Problem 14

Let S be the surface of revolution obtained by rotating the curve $y = 2\sqrt{x}$, $3 \le x \le 8$, about the x-axis. Compute the surface area of S.

A) $\frac{152\pi}{3}$ **B)** 523π **C)** $\frac{231\pi}{5}$ **D)** $\frac{315\pi}{2}$ **E)** $\frac{132\pi}{5}$ **F)** 325π
G) $\frac{251\pi}{3}$ **H)** $\frac{513\pi}{2}$

Problem 15

Which vector field do you see in the picture ?



A) $\vec{F}(x,y) = \langle\, x, y \,\rangle$ **B)** $\vec{F}(x,y) = \langle\, -x, y \,\rangle$ **C)** $\vec{F}(x,y) = \langle\, x, -y \,\rangle$
D) $\vec{F}(x,y) = \langle\, -x, -y \,\rangle$ **E)** $\vec{F}(x,y) = \langle\, y, x \,\rangle$ **F)** $\vec{F}(x,y) = \langle\, -y, x \,\rangle$
G) $\vec{F}(x,y) = \langle\, y, -x \,\rangle$ **H)** $\vec{F}(x,y) = \langle\, -y, -x \,\rangle$

The following problem will be hand-graded. To earn full credit you need to justify your answers.

Problem 16

a) Calculate

$$\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} 3 - \sqrt{x^2+y^2}\, \mathrm{d}y\, \mathrm{d}x.$$

b) Find the volume of the solid bounded by the cone $z = \sqrt{x^2+y^2}$, the cylinder $x^2 + y^2 = 9$ and the xy-plane.