Calculus III

Math 233 — Spring 2007

Final exam May 3rd.

This exam contains twenty problems numbered 1 through 20. All problems are multiple choice problems, and each counts 5% of your total score.

Problem 1

Find the center of the sphere $x^2 + y^2 + z^2 - 6x + 2y + 6 = 0$.

A) $(-6, 2, 0)$ **B)** $(-3, 1, 0)$ **C)** $(0, 0, 0)$ **D)** $(1, 1, 1)$ **E)** $(3, -1, 0)$
F) $(3, 1, 3)$ **G)** $(6, -2, 0)$ **H)** $(6, 2, 6)$

Problem 2

Compute

$$\text{curl } \langle 5x^2 + 3yz, 7y^2 + 2xz, 3z^2 + 3xy \rangle.$$

A) $\langle -3z, -2x, -3y \rangle$ **B)** $\langle -z, 0, x \rangle$ **C)** $\langle 10x, 14y, 6z \rangle$
D) $\langle 3xy - 2xz, 3yz - 3xy, 2xz - 3yz \rangle$ **E)** $\langle x, 0, -z \rangle$ **F)** $\langle 5x^2, 7y^2, 3z^2 \rangle$
G) $\langle 3x, 3y, 2z \rangle$ **H)** $\langle 3yz, 2xz, 3xy \rangle$

Problem 3

Calculate the divergence of $\langle 5x^2 + 3yz, 7y^2 + 2xz, 3z^2 + 3xy \rangle$.

A) $-2x - 3y - 3z$ **B)** $x - z$ **C)** $10x + 14y + 6z$ **D)** 0 **E)** 1
F) $5x^2 + 7y^2 + 3z^2$ **G)** $3x + 3y + 2z$ **H)** $3xy + 2xz + 3yz$

Problem 4

Let S be the part of the paraboloid $z = 3 - x^2 - y^2$ above the xy-plane, with upward orientation. Calculate the surface integral (flux integral) of

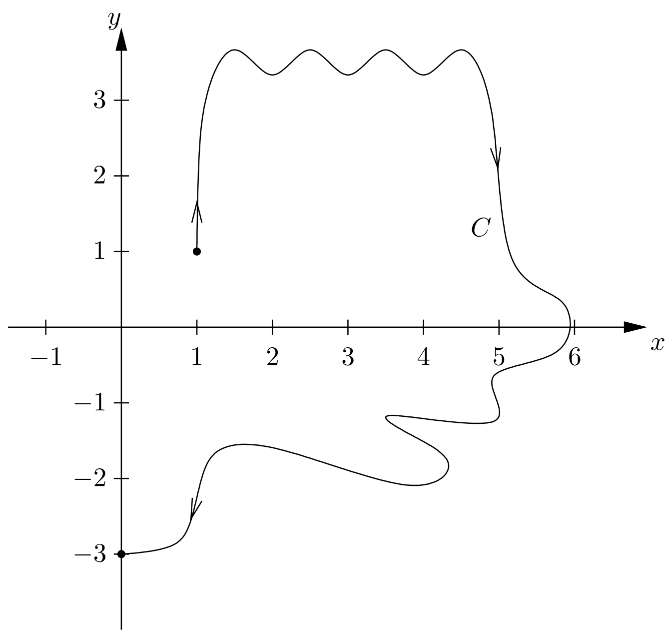
$$\vec{F}(x, y, z) = \langle x, y, -z \rangle$$

over S.

A) $\frac{\pi}{2}$ **B)** $\frac{3\pi}{2}$ **C)** $\frac{5\pi}{2}$ **D)** $\frac{7\pi}{2}$ **E)** $\frac{9\pi}{2}$ **F)** $\frac{11\pi}{2}$ **G)** $\frac{13\pi}{2}$
H) $\frac{15\pi}{2}$

Problem 5

Calculate $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F}(x, y) = \langle 2xy, x^2 + y \rangle$ and C is the curve in the figure below.



A) 0 **B)** -1 **C)** 2 **D)** 3 **E)** -4 **F)** -5 **G)** 6 **H)** -7

Problem 6

Let $\vec{a} = \langle 0, 1, -2\rangle$ and $\vec{b} = \langle -1, 2, -2\rangle$. Calculate

$$|\vec{a}|, \quad |\vec{b}|, \quad \vec{a}\cdot\vec{b}, \quad |\vec{a}\times\vec{b}|.$$

What is the largest value?

A) 0 **B)** 2 **C)** $\sqrt{5}$ **D)** $\sqrt{6}$ **E)** 3 **F)** 4 **G)** 5 **H)** 6

Problem 7

Find the arc length of the curve given by

$$\vec{r}(t) = \langle 2t\sqrt{t}, \cos(3t), \sin(3t)\rangle, \qquad 0 \le t \le 3.$$

A) 2 **B)** 6 **C)** 10 **D)** 14 **E)** 18 **F)** 22 **G)** 26 **H)** 30

Problem 8

What kind of surface is described by the parametrization

$$\vec{r}(u, v) = \langle u\cos v, u\sin v, u^2\rangle, \qquad 0 \le u \le 1,\ 0 \le v \le 2\pi?$$

A) Ellipsoid **B)** Paraboloid **C)** Plane **D)** Cone
E) Hyperboloid of one sheet **F)** Hyperboloid of two sheets **G)** Hemisphere
H) Cylinder

Problem 9

Evaluate

$$\int_0^4 \int_{\sqrt{y}}^2 \sqrt{1+x^3}\,\mathrm{d}x\,\mathrm{d}y$$

by reversing the order of integration.

A) $\frac{25}{9}$ **B)** $\frac{23}{9}$ **C)** $\frac{9}{52}$ **D)** $\frac{469}{52}$ **E)** $\frac{52}{9}$ **F)** $\frac{47}{9}$ **G)** $\frac{9}{25}$
H) $\frac{226}{25}$

Problem 10

Let $\vec{F}$ be the vector field

$$\vec{F}(x,y,z) = \langle x, e^{\sin(2z)}, (5+3y^{200})^7 \rangle.$$

Let E be the solid bounded by the cylinder $x^2+y^2=1$ and the planes $z=-2$ and $z=4$, while S is the (positively oriented) boundary surface of E.

Use the Divergence Theorem to evaluate

$$\iint_S \vec{F} \cdot \mathrm{d}\vec{S}.$$

A) 2π **B)** $2\sqrt{2}\pi$ **C)** 3π **D)** $2\sqrt{3}\pi$ **E)** 4π **F)** $3\sqrt{2}\pi$ **G)** 5π
H) 6π

Problem 11

Which of the following statements are true?

I) The vectors $\langle 1,3,-4 \rangle$ and $\langle 1,3,4 \rangle$ are orthogonal.

II) The vector $\langle 1,0,5 \rangle$ is longer than the vector $\langle 3,-3,1 \rangle$.

III) The cross product of $\langle 1,0,1 \rangle$ and $\langle 2,3,3 \rangle$ is parallel to $\langle 3,1,-3 \rangle$.

A) None of them **B)** Only I) **C)** Only II) **D)** Only III)
E) I) and II) **F)** I) and III) **G)** II) and III) **H)** All of them

Problem 12

Find the curvature of the curve

$$\vec{r}(t) = (\sin t)\vec{i} + (\cos t)\vec{j}, \qquad -\tfrac{\pi}{2} \le t \le \tfrac{\pi}{2},$$

at the unique point where the tangent to the curve is parallel to the x-axis.

A) 0 **B)** 1 **C)** 2 **D)** 3 **E)** 4 **F)** 5 **G)** 6 **H)** 7

Problem 13

Let D be the region in the xy-plane bounded by the upper semicircle $x^2 + y^2 = 1$ and the x-axis, and let C be the positively oriented boundary of D.

Use Green's Theorem to evaluate

$$\int_C \sqrt{x}e^{\cos x}\,\mathrm{d}x + \left(2x^4y + 4x^2y^3\right)\mathrm{d}y.$$

A) 0 **B)** 4 **C)** 8 **D)** 12 **E)** 16 **F)** 20 **G)** 24 **H)** 28

Problem 14

The space curve C is parametrized by

$$x = t^3,\ y = t,\ z = t^4, \qquad 0 \le t \le 1.$$

Evaluate

$$\int_C \left(3x + 8yz\right)\mathrm{d}s.$$

A) $\frac{1}{34}(26\sqrt{26}-1)$ **B)** $\frac{1}{24}(17\sqrt{17}-1)$ **C)** $\frac{1}{18}(26\sqrt{26}-1)$ **D)** $\frac{1}{13}(23\sqrt{23}-1)$
E) $\frac{1}{15}(17\sqrt{17}-1)$ **F)** $\frac{1}{16}(28\sqrt{28}-1)$ **G)** $\frac{1}{19}(15\sqrt{15}-1)$ **H)** $\frac{1}{13}(29\sqrt{29}-1)$

Problem 15

Find a constant M such that

$$\iint_R xe^y\,\mathrm{d}A = M \cdot \mathrm{area}(R),$$

where R is the rectangle

$$R = \left\{(x,y)\colon 2 \le x \le 4,\ 0 \le y \le 1\right\}.$$

A) $2\left(e-1\right)$ **B)** $2e$ **C)** $3\left(e-1\right)$ **D)** $3e$ **E)** $6\left(e-1\right)$ **F)** $6e$
G) $12\left(e-1\right)$ **H)** $12e$

Problem 16

Use Stokes' Theorem to evaluate

$$\int_C \vec{F} \cdot \mathrm{d}\vec{r},$$

where $\vec{F}(x, y, z) = \langle 0, 0, -x^2 \rangle$ and C is the boundary of the part of the plane

$$3x + 2y + z = 6$$

in the first octant. (C is oriented counterclockwise when viewed from above.)

A) 0 **B)** 4 **C)** 8 **D)** 12 **E)** 16 **F)** 20 **G)** 24 **H)** 28

Problem 17

Find the volume of the solid bounded by the cylinder $x^2 + y^2 = 6$, and the two planes $y + z = 7$ and $y + z = 14$.

A) 6π **B)** 7π **C)** 21π **D)** 36π **E)** 42π **F)** 63π **G)** 84π
H) 252π

Problem 18

Which of the vector fields

$$\langle x, y \rangle, \qquad \langle y^2 e^x, y^2 e^x \rangle, \qquad \langle yz, xz, xy \rangle$$

are conservative?

A) None of them **B)** Only $\langle x, y \rangle$ **C)** Only $\langle y^2 e^x, y^2 e^x \rangle$
D) Only $\langle yz, xz, xy \rangle$ **E)** $\langle x, y \rangle$ and $\langle y^2 e^x, y^2 e^x \rangle$ **F)** $\langle x, y \rangle$ and $\langle yz, xz, xy \rangle$
G) $\langle y^2 e^x, y^2 e^x \rangle$ and $\langle yz, xz, xy \rangle$ **H)** All of them

Problem 19

Given the function

$$f(x, y) = 5x^2y + \frac{5}{3}y^3 - 5x^2 - 5y^2 + 7.$$

How many local maxima, local minima, and saddle points does f have?

A) 1 local minimum, and 1 saddle point
B) 1 local maximum, and 1 saddle point
C) 1 local maximum, and 1 local minimum
D) 1 local maximum, 1 local minimum, and 2 saddle points
E) 1 local maximum, 2 local minima, and 1 saddle point
F) 2 local maxima, 1 local minimum, and 1 saddle point
G) 2 local minima, and 2 saddle points
H) 2 local maxima, and 2 saddle points

Problem 20

Evaluate

$$\iint_S (x + 2y)^2 \, \mathrm{d}S,$$

where S is the surface parametrized by

$$x = 2u + e^v, \; y = -u, \; z = 2v + 3, \qquad 0 \le u \le 3, \; 0 \le v \le \tfrac{1}{2}\ln(2).$$

A) $21\sqrt{21} - 20\sqrt{20}$ **B)** $22\sqrt{22} - 20\sqrt{20}$ **C)** $22\sqrt{22} - 21\sqrt{21}$
D) $23\sqrt{23} - 21\sqrt{21}$ **E)** $23\sqrt{23} - 22\sqrt{22}$ **F)** $24\sqrt{24} - 22\sqrt{22}$
G) $24\sqrt{24} - 23\sqrt{23}$ **H)** $25\sqrt{25} - 23\sqrt{23}$