Calculus III

Math 233 — Spring 2007

Practice exam April — Answers

This practice exam contains sixteen problems numbered 1 through 16. Problems 1 – 15 are multiple choice problems. Problem 16 is a free-response question.

Problem 1

Evaluate $\iint_R xy^2 \, \mathrm{d}A$ where R is the rectangle $R = \{(x, y) \colon -2 \leq x \leq 1, \ 0 \leq y \leq 1\}$.

A) -2 **B)** $\boxed{-\frac{1}{2}}$ **C)** 1 **D)** $\frac{3}{2}$ **E)** 3 **F)** 4 **G)** 10 **H)** 233

Problem 2

Find the area of the region bounded by the curves

$$y = x + 1, \qquad y = x - 1, \qquad y = \tfrac{1}{2}(x-1)^2 + \tfrac{1}{2}, \qquad \text{and} \qquad y = -\tfrac{1}{2}(x+1)^2 - \tfrac{1}{2}.$$

A) $\frac{2}{3}$ **B)** $\frac{4}{3}$ **C)** 2 **D)** $\boxed{\frac{8}{3}}$ **E)** $\frac{10}{3}$ **F)** 4 **G)** $\frac{14}{3}$ **H)** $\frac{16}{3}$

Problem 3

Let E be the sphere centered at the origin with radius 3. Evaluate

$$\iiint_E \frac{1}{\sqrt{x^2 + y^2}} \, \mathrm{d}V.$$

A) 9 **B)** 36 **C)** 3π **D)** 9π **E)** 36π **F)** $3\pi^2$ **G)** $\boxed{9\pi^2}$ **H)** $36\pi^2$

Problem 4

Use Lagrange multipliers to find the maximum value of the function

$$f(x, y, z) = xyz$$

on the sphere $x^2 + y^2 + z^2 = 12$.

A) -8 **B)** -4 **C)** -2 **D)** 0 **E)** 2 **F)** 4 **G)** 8 **H)** 12

Problem 5

Find the length of the tangent vector of the curve with parametric equations

$$x(t) = \int_2^t \frac{1}{\ln\theta}\,\mathrm{d}\theta, \qquad \text{and} \qquad y(t) = \int_2^t \ln\theta\,\mathrm{d}\theta$$

at the point corresponding to $t = e$.

A) $\frac{1}{2}$ **B)** $\frac{1}{2}\sqrt{2}$ **C)** 1 **D)** $\sqrt{2}$ **E)** $2(e-2)$ **F)** 2 **G)** e
H) $2e$

Problem 6

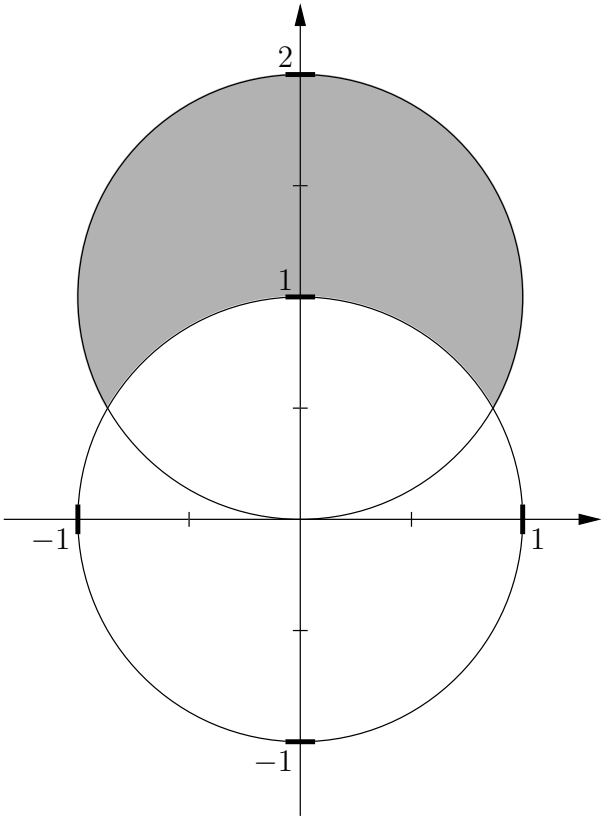
Let S be the surface of revolution obtained by rotating the curve $y = 2\sqrt{x}$, $3 \le x \le 8$, about the x-axis. Compute the tangent plane of S at the point $(4, 2\sqrt{2}, 2\sqrt{2})$.

A) $(x-4) - \sqrt{3}(y-2\sqrt{2}) + (z-2\sqrt{2}) = 0$
B) $\sqrt{2}(x-4) - (y-2\sqrt{2}) - \sqrt{2}(z-2\sqrt{2}) = 0$
C) $(x-4) + 3\sqrt{2}(y-2\sqrt{2}) + (z-2\sqrt{2}) = 0$
D) $(x-4) + (y-2\sqrt{2}) + (z-2\sqrt{2}) = 0$
E) $(x-4) - \sqrt{2}(y-2\sqrt{2}) - \sqrt{2}(z-2\sqrt{2}) = 0$
F) $-\sqrt{2}(y-2\sqrt{2}) - \sqrt{2}(z-2\sqrt{2}) = 0$
G) $2(x-4) + (y-2\sqrt{2}) - 2(z-2\sqrt{2}) = 0$
H) $4(x-4) + 2\sqrt{2}(y-2\sqrt{2}) + 2\sqrt{2}(z-2\sqrt{2}) = 0$

Problem 7

Let D be the gray region bounded by the polar curves in the figure below. Evaluate

$$\iint_D x\sqrt{x^2+y^2}\,\mathrm{d}A.$$



A) $\frac{\pi}{6}$ **B)** $\frac{5\pi}{6}-1$ **C)** 0 **D)** π **E)** 2 **F)** $\frac{2\pi}{3}$ **G)** $\frac{\pi}{3}$ **H)** $\frac{3}{2}$

Problem 8

Use the transformation

$$x = 3u, \quad y = 2v$$

to evaluate $\iint_R y^2 \, dA$, where R is the region bounded by the ellipse $4x^2 + 9y^2 = 36$.

Hint: You may want to use one of the identities

$$2\sin^2\theta = 1 - \cos 2\theta, \qquad 2\cos^2\theta = 1 + \cos 2\theta$$

when evaluating the integral.

A) $\frac{\pi}{2}$ **B)** 0 **C)** $\frac{\pi}{6}$ **D)** π **E)** 2π **F)** $\frac{7\pi}{4}$ **G)** 4π **H)** 6π

Problem 9

Compute the volume of the tetrahedron bounded by the four planes $y = 2x$, $y = -2x$, $5x + 3z = 15$, and $z = 0$.

A) 5 **B)** 6 **C)** 10 **D)** 15 **E)** 24 **F)** 30 **G)** 45 **H)** 48

Problem 10

Find the absolute maximum value of $f(x, y) = x^2 y$ on the disk $x^2 + y^2 \leq 4$.

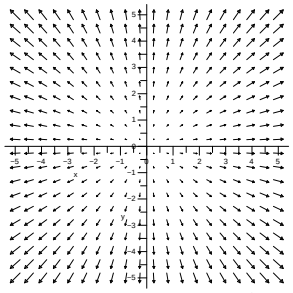
A) $\frac{\sqrt{3}}{2}$ **B)** 1 **C)** $\frac{\sqrt{13}}{2}$ **D)** $\frac{13}{4}$ **E)** $2\sqrt{3}$ **F)** $\frac{3}{8}$ **G)** $\frac{16\sqrt{3}}{9}$

H) $\frac{3}{4}$

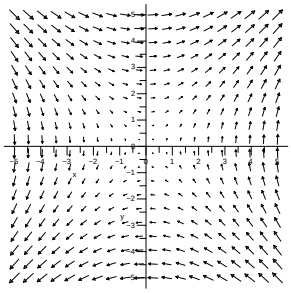
Problem 11

One (and only one) of the figures below depicts the vector field $\vec{F}(x, y) = \langle y, 3 - x \rangle$. Which one?

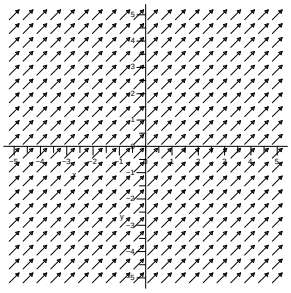
A)



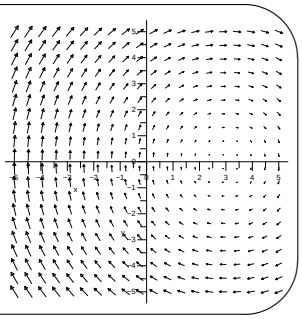
B)



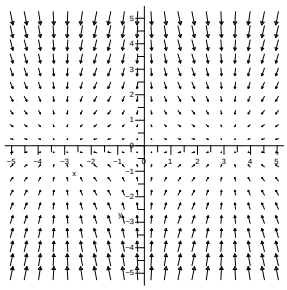
C)



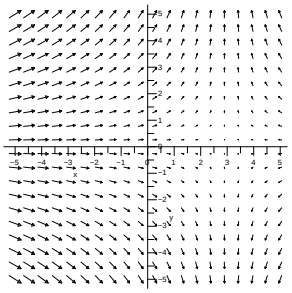
D)



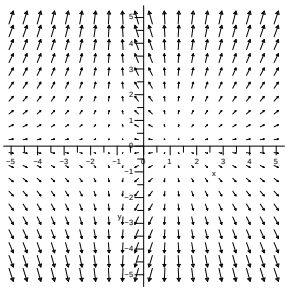
E)



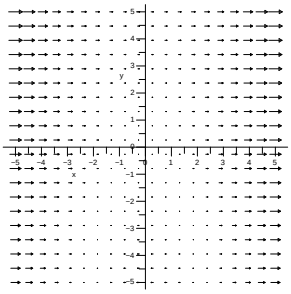
F)



G)



H)



Problem 12

Evaluate

$$\int_1^2 \int_1^{2/y} x(xy - 1)^{10} \, \mathrm{d}x \, \mathrm{d}y,$$

by changing the order of integration.

A) $\frac{1}{12}$ **B)** 1 **C)** $\frac{1}{11}$ **D)** $\frac{1}{13}$ **E)** 0 **F)** 11 **G)** $\frac{10}{11}$ **H)** $\frac{5}{12}$

Problem 13

Find the surface area of the part of the plane with vector equation

$$\vec{r}\,(u,v) \;= \langle 3+u, 4+v, 5u+9v\rangle$$

on the domain given by $0 \le u \le 3,\ 3 \le v \le 40$.

A) $2\sqrt{3}$ **B)** $\sqrt{107}$ **C)** $\sqrt{45}$ **D)** 37 **E)** $111\sqrt{107}$ **F)** $\sqrt{37}$
G) 107 **H)** $\sqrt{111}$

Problem 14

Evaluate $\iint_R e^{7-x^2-y^2}\,\mathrm{d}A$, where the domain R is given by

$$R = \{(x,y)\colon x \le 0, y \ge 0, 4 \le x^2+y^2 \le 9\}.$$

A) $e^7\pi$ **B)** $\frac{(e^3-e^{-2})\pi}{4}$ **C)** $\frac{(1-e)\pi}{2}$ **D)** $\frac{e^7}{5\pi}$ **E)** $\frac{e^3-e^{-2}}{\pi}$ **F)** e^9-e^4
G) $\frac{(1-e)\pi}{4}$ **H)** πe

Problem 15

Evaluate $\int_0^1\int_1^2\int_{1-x}^{1+x} y\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$.

A) $\frac{1}{4}$ **B)** $\frac{1}{2}$ **C)** $\frac{3}{4}$ **D)** 1 **E)** $\frac{5}{4}$ **F)** $\frac{3}{2}$ **G)** $\frac{7}{4}$ **H)** 2

The following problem is a free-response question. You should justify your answers.

Problem 16

a) Calculate

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} 4-x^2-y^2\,\mathrm{d}y\,\mathrm{d}x.$$

b) Let E be the solid bounded by the two paraboloids $x = 4-y^2-z^2$ and $x = y^2+z^2-4$. Find the volume of E.

a) 8π.

b) 16π.