Calculus III

Math 233 — Spring 2007

Practice exam May

This practice exam contains twenty problems numbered 1 through 20. All problems are multiple choice problems.

Problem 1

Compute $\langle 1,2,3\rangle \cdot \langle 4,5,6\rangle$.

A) 4 **B)** 8 **C)** 12 **D)** 16 **E)** 20 **F)** 24 **G)** 28 **H)** 32

Problem 2

Let $\vec{F}(x, y, z) = \langle x, y, xy \rangle$. Compute $\operatorname{curl} \vec{F}$ and $\operatorname{div} \vec{F}$.

A) $\operatorname{curl} \vec{F} = \langle 0, 0, xy \rangle$, $\operatorname{div} \vec{F} = 0$ **B)** $\operatorname{curl} \vec{F} = \langle 1, 1, 0 \rangle$, $\operatorname{div} \vec{F} = 0$
C) $\operatorname{curl} \vec{F} = \langle 0, 0, xy \rangle$, $\operatorname{div} \vec{F} = 1$ **D)** $\operatorname{curl} \vec{F} = \langle 1, 1, 0 \rangle$, $\operatorname{div} \vec{F} = 1$
E) $\operatorname{curl} \vec{F} = \langle x, -y, 0 \rangle$, $\operatorname{div} \vec{F} = 1$ **F)** $\operatorname{curl} \vec{F} = \langle 0, 0, xy \rangle$, $\operatorname{div} \vec{F} = 2$
G) $\operatorname{curl} \vec{F} = \langle 1, 1, 0 \rangle$, $\operatorname{div} \vec{F} = 2$ **H)** $\operatorname{curl} \vec{F} = \langle x, -y, 0 \rangle$, $\operatorname{div} \vec{F} = 2$

Problem 3

Let S be the graph of the function

$$g(x,y) = \cos x - e^y, \qquad 0 \le x \le \pi,\ 0 \le y \le 1.$$

Compute the surface integral (flux integral) of

$$\vec{F}(x,y,z) = \langle \sin x, e^{-y}, -\cos^2 x \rangle$$

over S.

A) $\frac{\pi}{2}$ **B)** $\frac{3\pi}{4}$ **C)** π **D)** $\frac{5\pi}{4}$ **E)** $\frac{3\pi}{2}$ **F)** $\frac{7\pi}{4}$ **G)** 2π **H)** $\frac{9\pi}{4}$

Problem 4

One of the diameters of a sphere S has endpoints $(5, 3, 7)$ and $(-1, 3, -1)$. Find an equation for S.

A) $(x+3)^2 + (y+5)^2 + (z+5)^2 = 4$ **B)** $(x-1)^2 + (y-1)^2 + (z-4)^2 = 1$
C) $(x-4)^2 + (y-5)^2 + (z+4)^2 = 9$ **D)** $(x+4)^2 + (y-3)^2 + (z+1)^2 = 0$
E) $(x-2)^2 + (y-3)^2 + (z-3)^2 = 25$ **F)** $(x-1)^2 + (y+2)^2 + (z+3)^2 = 1$
G) $(x-3)^2 + (y-4)^2 + (z-5)^2 = 9$ **H)** $(x+5)^2 + (y-4)^2 + (z+4)^2 = 4$

Problem 5

What kind of surface is described by the parametrization

$$\vec{r}(u,v) = \langle u\cos v, u\sin v, u\rangle, \qquad 0 \le u \le 1,\ 0 \le v \le 2\pi?$$

A) Ellipsoid **B)** Paraboloid **C)** Plane **D)** Cone
E) Hyperboloid of one sheet **F)** Hyperboloid of two sheets **G)** Hemisphere
H) Cylinder

Problem 6

Find the arc length of the polar curve $r = 4\sin\theta$ from $\theta = 0$ to $\theta = \pi$.

A) 0 **B)** π **C)** $\pi\sqrt{2}$ **D)** 2π **E)** $2\sqrt{2}\pi$ **F)** 3π **G)** 4π
H) 8π

Problem 7

The curve C consists of the line segment from $(3, 1, 4)$ to $(1, 5, 9)$ followed by the line segment from $(1, 5, 9)$ to $(2, 6, 5)$. Use the Fundamental Theorem for Line Integrals to compute

$$\int_C yz\,\mathrm{d}x + xz\,\mathrm{d}y + xy\,\mathrm{d}z.$$

A) 12 **B)** 15 **C)** 33 **D)** 45 **E)** 48 **F)** 57 **G)** 60 **H)** 72

Problem 8

Find the minimum value of

$$f(x, y) = (x - 1)^2 + (y - 4)^2 + (3 - x - 2y)^2.$$

A) 0 **B)** 1 **C)** 3 **D)** 6 **E)** 8 **F)** 12 **G)** 16 **H)** 20

Problem 9

Find the distance from the point $(1, 4, 4)$ to the plane $x + 2y + z = 7$.

A) 0 **B)** 1 **C)** $\sqrt{2}$ **D)** 2 **E)** $\sqrt{6}$ **F)** $2\sqrt{3}$ **G)** 4
H) $2\sqrt{5}$

Problem 10

Find the volume of the solid that lies below the plane $x + y + z = 7$ and above the triangle in the xy-plane with vertices $(1, 1)$, $(1, 3)$ and $(2, 2)$.

A) $\frac{11}{3}$ **B)** $\frac{13}{3}$ **C)** 5 **D)** $\frac{17}{3}$ **E)** $\frac{19}{3}$ **F)** 7 **G)** $\frac{23}{3}$ **H)** $\frac{25}{3}$

Problem 11

Let S be the closed surface bounding the cube

$$E = \left\{(x, y, z) \colon 0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le 1\right\}.$$

Use the Divergence Theorem to compute

$$\iint_S \vec{F} \cdot \mathrm{d}\vec{S},$$

where $\vec{F}(x, y, z) = \big(x + yz\big)\vec{i} + \big(y + xz\big)\vec{j} + \big(z + xy\big)\vec{k}$.

A) 0 **B)** 1 **C)** 2 **D)** 3 **E)** 4 **F)** 5 **G)** 6 **H)** 7

Problem 12

Let $\vec{F}(x, y) = \langle y - e^{\cos x} \sin x, x \rangle$. Let f be such that $\vec{\nabla} f = \vec{F}$ and $f(0, 0) = e$.

Calculate $f(\pi, \frac{1}{\pi})$.

A) $\frac{1}{e}$ **B)** e **C)** $\frac{1}{e} + 1$ **D)** $e + 1$ **E)** $\frac{1}{\pi}$ **F)** π **G)** $\frac{1}{\pi} + 1$
H) $\pi + 1$

Problem 13

Let S be the part of the paraboloid $z = x^2 + y^2$ between the planes $z = 1$ and $z = 9$. Evaluate

$$\iint_S \frac{z}{\sqrt{1+4z}} \, \mathrm{d}S.$$

A) 5π **B)** 10π **C)** 15π **D)** 20π **E)** 25π **F)** 30π **G)** 35π
H) 40π

Problem 14

Let C be the closed curve consisting of the line segment from $(1,0)$ to $(-1,0)$, and the semicircle parametrized by

$$x(t) = -\cos t,\ y(t) = -\sin t, \qquad 0 \leq t \leq \pi.$$

Use Green's Theorem to compute

$$\int_C -y\,\mathrm{d}x + x\,\mathrm{d}y.$$

A) 0 **B)** $\frac{\pi}{2}$ **C)** π **D)** $\frac{3\pi}{2}$ **E)** 2π **F)** 3π **G)** 4π **H)** 5π

Problem 15

Let E be the part of the solid bounded by the sphere $x^2 + y^2 + z^2 = 4$ that lies in the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$). Evaluate

$$\iiint_E 2z \, \mathrm{d}V.$$

A) $\frac{\pi}{4}$ **B)** $\frac{\pi}{2}$ **C)** $\frac{3\pi}{4}$ **D)** π **E)** $\frac{5\pi}{4}$ **F)** $\frac{3\pi}{2}$ **G)** $\frac{7\pi}{4}$ **H)** 2π

Problem 16

Consider the vector field $\vec{F}(x, y, z) = \langle -z, y, x \rangle$ defined on the space curve C parametrized by

$$\vec{r}(t) = \langle t, t^2, t^3 \rangle, \qquad 2 \leq t \leq 3.$$

Evaluate the line integral $\int_C \vec{F} \cdot \mathrm{d}\vec{r}$.

A) 28 **B)** 34 **C)** 45 **D)** 63 **E)** 65 **F)** 72 **G)** 76 **H)** 80

Problem 17

Find the constant K such that

$$\iint_D K \, \mathrm{d}A = 233,$$

where D is the disk $D = \{(x, y) \colon x^2 + y^2 \leq 233\}$.

A) $\frac{1}{233\pi}$ **B)** $\frac{1}{233}$ **C)** $\frac{1}{\pi}$ **D)** 1 **E)** π **F)** $\frac{233}{\pi}$ **G)** 233
H) 233π

Problem 18

Let S be the part of the cone $z = 4 - \sqrt{x^2 + y^2}$ that lies above the xy-plane, with upward orientation. Use Stokes' Theorem to calculate

$$\iint_S \operatorname{curl} \vec{F} \cdot \mathrm{d}\vec{S},$$

where $\vec{F}(x, y, z) = \langle x^2 + y^2, ze^{xy}, xye^z \rangle$.

A) 0 **B)** 16 **C)** 32 **D)** 48 **E)** 64 **F)** 80 **G)** 96 **H)** 112

Problem 19

Let C be the space curve given by

$$\vec{r}(t) = \langle \cos t, \sin t, t \rangle, \qquad 0 \leq t \leq \pi.$$

Evaluate

$$\int_C \big(xy + z\big)\, \mathrm{d}s.$$

A) $\sqrt{2}\pi$ **B)** $2\sqrt{2}\pi$ **C)** $\frac{1}{2}\sqrt{2}\pi^2$ **D)** $\sqrt{2}\pi^2$ **E)** $1+\sqrt{2}\pi$ **F)** $1+2\sqrt{2}\pi$
G) $1 + \frac{1}{2}\sqrt{2}\pi^2$ **H)** $1 + \sqrt{2}\pi^2$

Problem 20

Let

$$f(x, y) = (x - y)^{2007},$$

where $x(u, v) = uv$ and $y(u, v) = \frac{u}{v}$.

Calculate $\frac{\partial f}{\partial u}$ at the point where $u = \frac{2}{3}$ and $v = 2$.

A) 0 **B)** $2007 \cdot \frac{1}{2}$ **C)** 2007 **D)** $2007 \cdot \frac{3}{2}$ **E)** $2007 \cdot 2$ **F)** $2007 \cdot \frac{5}{2}$
G) $2007 \cdot 3$ **H)** $2007 \cdot \frac{7}{2}$