# Calculus III

## Math 233 — Spring 2007

## Practice exam May — Answers

This practice exam contains twenty problems numbered 1 through 20. All problems are multiple choice problems.

### Problem 1

Compute $\langle 1, 2, 3\rangle \cdot \langle 4, 5, 6\rangle$.

**A)** 4 **B)** 8 **C)** 12 **D)** 16 **E)** 20 **F)** 24 **G)** 28 $\boxed{\textbf{H)}\ 32}$

### Problem 2

Let $\vec{F}(x, y, z) = \langle x, y, xy\rangle$. Compute $\operatorname{curl}\vec{F}$ and $\operatorname{div}\vec{F}$.

**A)** $\operatorname{curl}\vec{F} = \langle 0, 0, xy\rangle$, $\operatorname{div}\vec{F} = 0$ **B)** $\operatorname{curl}\vec{F} = \langle 1, 1, 0\rangle$, $\operatorname{div}\vec{F} = 0$
**C)** $\operatorname{curl}\vec{F} = \langle 0, 0, xy\rangle$, $\operatorname{div}\vec{F} = 1$ **D)** $\operatorname{curl}\vec{F} = \langle 1, 1, 0\rangle$, $\operatorname{div}\vec{F} = 1$
**E)** $\operatorname{curl}\vec{F} = \langle x, -y, 0\rangle$, $\operatorname{div}\vec{F} = 1$ **F)** $\operatorname{curl}\vec{F} = \langle 0, 0, xy\rangle$, $\operatorname{div}\vec{F} = 2$
**G)** $\operatorname{curl}\vec{F} = \langle 1, 1, 0\rangle$, $\operatorname{div}\vec{F} = 2$ $\boxed{\textbf{H)}\ \operatorname{curl}\vec{F} = \langle x, -y, 0\rangle,\quad \operatorname{div}\vec{F} = 2}$

## Problem 3

Let $S$ be the graph of the function

$$g(x,y) = \cos x - e^y, \qquad 0 \le x \le \pi,\ 0 \le y \le 1.$$

Compute the surface integral (flux integral) of

$$\vec{F}(x,y,z) = \langle \sin x, e^{-y}, -\cos^2 x \rangle$$

over $S$.

**A)** $\frac{\pi}{2}$ **B)** $\frac{3\pi}{4}$ $\boxed{\textbf{C)}\ \pi}$ **D)** $\frac{5\pi}{4}$ **E)** $\frac{3\pi}{2}$ **F)** $\frac{7\pi}{4}$ **G)** $2\pi$ **H)** $\frac{9\pi}{4}$

## Problem 4

One of the diameters of a sphere $S$ has endpoints $(5,3,7)$ and $(-1,3,-1)$. Find an equation for $S$.

**A)** $(x+3)^2 + (y+5)^2 + (z+5)^2 = 4$ **B)** $(x-1)^2 + (y-1)^2 + (z-4)^2 = 1$
**C)** $(x-4)^2 + (y-5)^2 + (z+4)^2 = 9$ **D)** $(x+4)^2 + (y-3)^2 + (z+1)^2 = 0$
$\boxed{\textbf{E)}\ (x-2)^2 + (y-3)^2 + (z-3)^2 = 25}$ **F)** $(x-1)^2 + (y+2)^2 + (z+3)^2 = 1$
**G)** $(x-3)^2 + (y-4)^2 + (z-5)^2 = 9$ **H)** $(x+5)^2 + (y-4)^2 + (z+4)^2 = 4$

## Problem 5

What kind of surface is described by the parametrization

$$\vec{r}(u,v) = \langle u\cos v, u\sin v, u \rangle, \qquad 0 \le u \le 1,\ 0 \le v \le 2\pi?$$

**A)** Ellipsoid **B)** Paraboloid **C)** Plane $\boxed{\textbf{D)}\ \text{Cone}}$
**E)** Hyperboloid of one sheet **F)** Hyperboloid of two sheets **G)** Hemisphere
**H)** Cylinder

## Problem 6

Find the arc length of the polar curve $r = 4\sin\theta$ from $\theta = 0$ to $\theta = \pi$.

**A)** $0$ **B)** $\pi$ **C)** $\pi\sqrt{2}$ **D)** $2\pi$ **E)** $2\sqrt{2}\pi$ **F)** $3\pi$ $\boxed{\textbf{G)}\ 4\pi}$
**H)** $8\pi$

## Problem 7

The curve $C$ consists of the line segment from $(3, 1, 4)$ to $(1, 5, 9)$ followed by the line segment from $(1, 5, 9)$ to $(2, 6, 5)$. Use the Fundamental Theorem for Line Integrals to compute

$$\int_C yz\,\mathrm{d}x + xz\,\mathrm{d}y + xy\,\mathrm{d}z.$$

**A)** 12 **B)** 15 **C)** 33 **D)** 45 $\boxed{\textbf{E)}\ 48}$ **F)** 57 **G)** 60 **H)** 72

## Problem 8

Find the minimum value of

$$f(x, y) = (x-1)^2 + (y-4)^2 + (3-x-2y)^2.$$

**A)** 0 **B)** 1 **C)** 3 $\boxed{\textbf{D)}\ 6}$ **E)** 8 **F)** 12 **G)** 16 **H)** 20

## Problem 9

Find the distance from the point $(1, 4, 4)$ to the plane $x + 2y + z = 7$.

**A)** 0 **B)** 1 **C)** $\sqrt{2}$ **D)** 2 $\boxed{\textbf{E)}\ \sqrt{6}}$ **F)** $2\sqrt{3}$ **G)** 4
**H)** $2\sqrt{5}$

## Problem 10

Find the volume of the solid that lies below the plane $x + y + z = 7$ and above the triangle in the $xy$-plane with vertices $(1, 1)$, $(1, 3)$ and $(2, 2)$.

$\boxed{\textbf{A)}\ \frac{11}{3}}$ **B)** $\frac{13}{3}$ **C)** 5 **D)** $\frac{17}{3}$ **E)** $\frac{19}{3}$ **F)** 7 **G)** $\frac{23}{3}$ **H)** $\frac{25}{3}$

## Problem 11

Let $S$ be the closed surface bounding the cube

$$E = \{(x, y, z) \colon 0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le 1\}.$$

Use the Divergence Theorem to compute

$$\iint_S \vec{F} \cdot \mathrm{d}\vec{S},$$

where $\vec{F}(x, y, z) = (x + yz)\vec{\imath} + (y + xz)\vec{\jmath} + (z + xy)\vec{k}$.

**A)** 0 **B)** 1 **C)** 2 $\boxed{\textbf{D)}\ 3}$ **E)** 4 **F)** 5 **G)** 6 **H)** 7

## Problem 12

Let $\vec{F}(x, y) = \langle y - e^{\cos x} \sin x, x \rangle$. Let $f$ be such that $\vec{\nabla} f = \vec{F}$ and $f(0, 0) = e$.

Calculate $f(\pi, \frac{1}{\pi})$.

**A)** $\frac{1}{e}$ **B)** $e$ $\boxed{\textbf{C)}\ \frac{1}{e} + 1}$ **D)** $e + 1$ **E)** $\frac{1}{\pi}$ **F)** $\pi$ **G)** $\frac{1}{\pi} + 1$
**H)** $\pi + 1$

## Problem 13

Let $S$ be the part of the paraboloid $z = x^2 + y^2$ between the planes $z = 1$ and $z = 9$. Evaluate

$$\iint_S \frac{z}{\sqrt{1 + 4z}}\, \mathrm{d}S.$$

**A)** $5\pi$ **B)** $10\pi$ **C)** $15\pi$ **D)** $20\pi$ **E)** $25\pi$ **F)** $30\pi$ **G)** $35\pi$
$\boxed{\textbf{H)}\ 40\pi}$

## Problem 14

Let $C$ be the closed curve consisting of the line segment from $(1, 0)$ to $(-1, 0)$, and the semicircle parametrized by

$$x(t) = -\cos t,\ y(t) = -\sin t, \qquad 0 \leq t \leq \pi.$$

Use Green's Theorem to compute

$$\int_C -y\,\mathrm{d}x + x\,\mathrm{d}y.$$

**A)** $0$ **B)** $\frac{\pi}{2}$ $\boxed{\textbf{C)}\ \pi}$ **D)** $\frac{3\pi}{2}$ **E)** $2\pi$ **F)** $3\pi$ **G)** $4\pi$ **H)** $5\pi$

## Problem 15

Let $E$ be the part of the solid bounded by the sphere $x^2 + y^2 + z^2 = 4$ that lies in the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$). Evaluate

$$\iiint_E 2z\,\mathrm{d}V.$$

**A)** $\frac{\pi}{4}$ **B)** $\frac{\pi}{2}$ **C)** $\frac{3\pi}{4}$ **D)** $\pi$ **E)** $\frac{5\pi}{4}$ **F)** $\frac{3\pi}{2}$ **G)** $\frac{7\pi}{4}$ $\boxed{\textbf{H)}\ 2\pi}$

## Problem 16

Consider the vector field $\vec{F}(x, y, z) = \langle -z, y, x\rangle$ defined on the space curve $C$ parametrized by

$$\vec{r}(t) = \langle t, t^2, t^3\rangle, \qquad 2 \leq t \leq 3.$$

Evaluate the line integral $\int_C \vec{F} \cdot \mathrm{d}\vec{r}$.

**A)** 28 **B)** 34 **C)** 45 **D)** 63 $\boxed{\textbf{E)}\ 65}$ **F)** 72 **G)** 76 **H)** 80

## Problem 17

Find the constant $K$ such that

$$\iint_D K \, \mathrm{d}A = 233,$$

where $D$ is the disk $D = \{(x, y) \colon x^2 + y^2 \leq 233\}$.

**A)** $\frac{1}{233\pi}$ **B)** $\frac{1}{233}$ $\boxed{\textbf{C)}\ \frac{1}{\pi}}$ **D)** 1 **E)** $\pi$ **F)** $\frac{233}{\pi}$ **G)** 233
**H)** $233\pi$

## Problem 18

Let $S$ be the part of the cone $z = 4 - \sqrt{x^2 + y^2}$ that lies above the $xy$-plane, with upward orientation. Use Stokes' Theorem to calculate

$$\iint_S \operatorname{curl} \vec{F} \cdot \mathrm{d}\vec{S},$$

where $\vec{F}(x, y, z) = \langle x^2 + y^2, ze^{xy}, xye^z \rangle$.

$\boxed{\textbf{A)}\ 0}$ **B)** 16 **C)** 32 **D)** 48 **E)** 64 **F)** 80 **G)** 96 **H)** 112

## Problem 19

Let $C$ be the space curve given by

$$\vec{r}(t) = \langle \cos t, \sin t, t \rangle, \qquad 0 \leq t \leq \pi.$$

Evaluate

$$\int_C (xy + z) \, \mathrm{d}s.$$

**A)** $\sqrt{2}\pi$ **B)** $2\sqrt{2}\pi$ $\boxed{\textbf{C)}\ \frac{1}{2}\sqrt{2}\pi^2}$ **D)** $\sqrt{2}\pi^2$ **E)** $1 + \sqrt{2}\pi$ **F)** $1 + 2\sqrt{2}\pi$
**G)** $1 + \frac{1}{2}\sqrt{2}\pi^2$ **H)** $1 + \sqrt{2}\pi^2$

## Problem 20

Let

$$f(x,y) = (x-y)^{2007},$$

where $x(u,v) = uv$ and $y(u,v) = \frac{u}{v}$.

Calculate $\frac{\partial f}{\partial u}$ at the point where $u = \frac{2}{3}$ and $v = 2$.

**A)** $0$ **B)** $2007 \cdot \frac{1}{2}$ **C)** $2007$ $\boxed{\textbf{D)}\ 2007 \cdot \frac{3}{2}}$ **E)** $2007 \cdot 2$ **F)** $2007 \cdot \frac{5}{2}$
**G)** $2007 \cdot 3$ **H)** $2007 \cdot \frac{7}{2}$