# Elementary geometry from an advanced point of view

Math 302 — Fall 2008

Final exam 12/18

You should answer 9 problems. Each problem gives equal credit.

**Part I:** Answer 3 of the 7 problems A, B, C, D, E, F, and G.

### Problem A

Show that if a line intersects a triangle, then the line intersects at least two sides of the triangle.

### Problem B

Prove that if $D$ is in the interior of $\angle BAC$, then $\overrightarrow{AD}$ intersects $\overline{BC}$, in a point between $B$ and $C$.

### Problem C

Show that if ASA is taken as a postulate instead of SAS, then SAS can be proved as a theorem.

### Problem D

Prove SSS: *Given $\triangle ABC$ and $\triangle DEF$. If $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$ and $\overline{BC} \cong \overline{EF}$, then $\triangle ABC \cong \triangle DEF$.*

### Problem E

Show that the upper base angles of a Saccheri quadrilateral are congruent.

### Problem F

Show that the upper base of a Saccheri quadrilateral is at least as long as its lower base.

### Problem G

Prove that in euclidean geometry, the angle sum of a triangle is 180.

**Part II:** Answer all 6 of the problems 1a, 1b, 2, 3, 4a and 4b.

## Problem 1

**a)** Define similar triangles.

**b)** Prove the AAA similarity theorem.

## Problem 2

Prove the exterior angle theorem: *Given* $\triangle ABC$*. If* $A$-$C$-$D$*, then* $\angle BCD > \angle B$.

## Problem 3

Draw (sketch) two Saccheri quadrilaterals in the (upper) hyperbolic halfplane. One where the upper base angles are close to being right angles, and one where the upper base angles are very small.

## Problem 4

**a)** Show that if $D$ is in the interior of $\triangle ABC$, then $BD + DC < BA + AC$ and $\angle BDC > \angle BAC$.

**b)** Let $\triangle ABC$ be a right triangle, with the right angle at $C$. Let $E$ be such that $B$-$E$-$C$. Show that there is a point $D$ in the interior of $\triangle ABC$ so that $BD + DE > BA + AC$.