

Differential equations

Math 217 — Fall 2009

Sample October

This practice exam contains fourteen problems numbered 1 through 13. Problems 1 – 12 are multiple choice problems. Problem 13 is a free-response question.

Problem 1

Suppose $y = e^{-x} \sin 2x$ is a solution to the constant coefficient homogeneous differential equation

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y' + a_0 y = 0$$

where a_n, \dots, a_0 are real numbers. Which of the following must be true?

- A) $r = 2 - i$ is a root of the characteristic equation.
- B) $y = e^x \sin 2x$ is a solution to the differential equation.
- C) $y = \cos 2x$ is a solution to the differential equation.
- D) $r = 1 + 2i$ is a root of the characteristic equation.
- E) $y = e^{2x} \cos x$ is a solution of the differential equation.
- F) $y = e^{-x} \cos 2x$ is a solution of the differential equation.

Problem 2

Suppose y_1 and y_2 are continuously differentiable functions on the real line and $W(y_1, y_2)(x) = x^4$, where W is the Wronskian. Which of the following are true.

- I) y_1 and y_2 are the solutions to the second-order linear differential equation $y'' + xy' + \sin(x)y = 0$ on the real line.
 - II) y_1 and y_2 are linearly independent on the real line.
 - III) $W(y_2, y_1) = -x^4$.
- A) Only I B) Only II C) Only III D) I and II E) I and II
F) II and III

Problem 3

An arrow is shot straight upward from the ground with initial velocity of 160 ft/s. It experiences both the deceleration of gravity and deceleration $v^2/800$ due to air resistance. How long does it take to reach its maximum height?

- A) 5 s B) 4.51 s C) 0.78 s D) 4.92 s E) 3.92 s F) 2.38 s

Problem 4

Let $y(x)$ be the solution of the initial value problem

$$\frac{dy}{dx} = x + y, \quad y(0) = 1.$$

Use improved Euler method to compute an approximation to $y(0.2)$ with $h = 0.1$.

- A) 1.20 B) 1.21 C) 1.22 D) 1.23 E) 1.24 F) 1.25

Problem 5

Suppose $y_p = q(x)$ is a solution of the differential equation $y'' + 2y' + 1 = f(x)$. Which of the following represents a general solution.

- A) $y = c_1e^{-x} + c_2xe^{-x}$.
B) $y = c_1 + e^{-2x} + c_2q(x)$.
C) $y = c_1e^{-x} + c_2xe^{-x} + q(x)$.
D) $y = c_1 + c_2xe^{-2x} + q(x)$.
E) $y = c_1e^{-x} + c_2q(x)$.
F) $y = c_1e^{-x} + c_2e^{-2x} + q(x)$.

Problem 6

Which of the following functions is a particular solution of

$$y''' - 4y' = 8t + 15 \cos t + 8e^{-2t}?$$

- A) t B) e^{-2t} C) te^{-2t} D) $-3 \sin t$ E) $-t^2$ F) $-t^2 - 3 \sin t + te^{-2t}$

Problem 7

Find the characteristic equation for the associated homogeneous equation to

$$y^{(4)} + 3y'' - y' + 2y = x.$$

- A) $r^4 + 3r^3 - r + 2 = 0$ B) $r^4 + 3r^3 - r + 2 = x$ C) $r^4 + 3r^2 - r + 2 = 0$
D) $r^4 + 3r^2 - r + 2 = x$ E) $r^4 + 3r^3 - r + 1 = 0$ F) $r^4 + 3r^2 - r + 1 = x$

Problem 8

Suppose that a motorboat is moving at 50 ft/s when its motor is turned off and that 10 s later the boat has slowed to 25 ft/s. Assume that the resistance the boat encounters while coasting is proportional to its velocity. How far will the boat coast in all?

- A) 503.1 ft
B) 652.6 ft
C) 721.3 ft
D) 1000 ft
E) 256.1 ft
F) 14.4 ft

Problem 9

Solve the following initial value problem

$$y'' + 2y' + y = 0, \quad y(0) = -1, \quad y'(0) = 2.$$

- A) $y = \frac{1}{2}e^x - \frac{3}{2}e^{-x}$ B) $y = \frac{3}{2}e^x - \frac{1}{2}e^{-x}$ C) $y = e^x - 3xe^{-x}$ D) $y = -e^{-x} + xe^{-x}$
E) $y = 3xe^{-x} - e^{-x}$ F) $y = e^x - xe^x$

Problem 10

Suppose a body with mass .5 kg is attached to a spring with spring constant $k = 32$ N/m and that no other forces act on the body. The body is set in motion by compressing the spring by 2 m to the left of its equilibrium position and applying an initial velocity of -16 m/s. What is the farthest distance the body will ever be from its equilibrium position?

- A) $\sqrt{2}$ m B) $2\sqrt{2}$ m C) $\sqrt{5}$ m D) 2 m E) 4 m F) 8 m

Problem 11

Find a particular solution to the differential equation

$$y'' + y = xe^x$$

- A) $2e^x$ B) $2xe^x$ C) $2xe^x - e^x$ D) $2e^x - xe^x$ E) $xe^x - 2e^x$ F) xe^x

Problem 12

Which of the following sets of functions is linearly independent on the real line?

- A) $1, \cos^2(2x), \sin^2(2x)$ B) $x^2 - x, x^2, x$ C) $0, 1, x$ D) $x, \sin(x), \cos(x)$
E) $x, -x, |x|$ F) $0, x^2, \arctan(x)$

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The following problem is a free-response question. You should justify your answers.

Problem 13

- a) Show that $x_1(t) = e^{-2t} \cos(t)$ and $x_2(t) = e^{-2t} \sin(t)$ are solutions of

$$x'' + 4x' + 5x = 0.$$

Compute their Wronskian.

- b) Consider a damped mass-spring-dashpot system with a mass of 1 kg, a spring with constant $k = 5$, and the dashpot providing 4 N of resistance for each meter per second of velocity. Is the system overdamped, critically damped, or underdamped?
- c) Now suppose an external force acts on the mass-spring-dashpot system from part b) providing an external force of $F_E(t) = 5 \cos(2t)$, thus giving the differential equation

$$x'' + 4x' + 5x = 5 \cos(2t)$$

describing the system. Find the general solution to this differential equation.