

Math 331 Spring 2006
Midterm 1 (By Feb.27th 2:00)

In each number choose either (a) or (b) and show your work in detail.
In each question, (a) and (b) are 20 points and 25 points respectively.

1. (a) Find a formula for

$$\sum_{k=0}^n k^2 \binom{n}{k} = ?$$

- (b) Prove that

$$\sum_{k=0}^n \binom{n}{k}^2 = \binom{2n}{n}.$$

2. (a) Let A be the set of all 7-th root of unity. Consider a function

$$f : A \rightarrow A \\ \alpha \mapsto \alpha^3.$$

Is this bijective ?

- (b) Let B be the set of all 101-th root of unity. Consider a function

$$f : B \rightarrow B \\ \alpha \mapsto \alpha^7.$$

Is this bijective ?

3. (a) Consider a permutation in f in S_{11} defined to satisfy

$$e^{\frac{2\pi i}{11} f(j)} = e^{\frac{2\pi i}{11} j^3} \text{ for each } j = 1, \dots, 11.$$

Find its parity.

- (b) How many permutation in S_n has positive signature (positive parity) ? Justify your answer.

4. (a) Find the smallest positive integer which leaves remainder 4, 6, 8 after dividing by 5, 7, 9, respectively.

- (b) Solve the linear system

$$x \equiv 12 \pmod{25} \\ x \equiv 20 \pmod{30}.$$

5. (a) Is it possible a group can have only two elements ? Justify your answer.

- (b) Suppose G is a group with 3 elements, is this abelian ? Justify your answer.