Math 429 Fall 2005
Midterm 2 (By Nov.28th 12:00)

1. (30 points) Let A be any element in $Mat_{n\times n}(\mathbb{R})$ where $A+I_n$ is non-singular. Define an operation T as $T(A)=(I_n-A)(A+I_n)^{-1}$.

   Note for a $n\times n$-matrix B, B is called orthogonal (resp. skew-symmetric) if $BB^T=I_n$ (resp. $B^T=-B$) .

   (1) Show that $T(T(A))=A$.

   (2) Show if A is an orthogonal matrix then $T(A)$ is skew-symmetric. And also show if A is skew-symmetric then $T(A)$ is orthogonal.

2. (20 points) Find the determinant of the matrix

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ a_1 & a_2 & a_3 & a_4 \\ a_1^2 & a_2^2 & a_3^2 & a_4^2 \\ (a_2+a_3+a_4)^3 & (a_1+a_3+a_4)^3 & (a_1+a_2+a_4)^3 & (a_1+a_2+a_3)^3 \end{pmatrix}.$$

3. (20 points) Consider a linear transformation A: $\mathbb{R}^{2005}\to\mathbb{R}^{2005}$ defined as

$$A=\begin{pmatrix} 1 & 2 & 3 & ... & 2005 \\ 2 & 2^2 & 2\times 3 & ... & 2\times 2005 \\ 3 & 3\times 2 & 3^2 & ... & 3\times 2005 \\ . & . & . & ... & . \\ 2005 & 2005\times 2 & 2005\times 3 & ... & 2005^2 \end{pmatrix}.$$

   Find eigenvalues of A and determine if A is diagonalizable.

4. (30 points) Let A be an invertible real $n\times n$ matrix and its characteristic polynomial $f(x)$ be the same with its minimal polynomial. Find the minimal polynomial of A^{-1}. (Hint. The expression of answer contains $f(x^{-1})$.)