

**Math 331 Spring 2006**  
**Assignment 1: Due by Jan 30**

1. Prove that if  $a_1, a_2, \dots, a_n$  are positive numbers, then

$$(a_1 + a_2 + \dots + a_n) \left( \frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n} \right) \geq n^2.$$

And find the minimum of

$$(b_1 + 2b_2 + 3b_3) \left( \frac{3}{b_1} + \frac{2}{b_2} + \frac{1}{b_3} \right)$$

where  $b_1, b_2$  and  $b_3$  are positive numbers.

2. Prove that  $4^{n+1} + 5^{2n-1}$  is divisible by 21 for all  $n \geq 1$ .
3. Prove that if  $n \geq 2$ , then

$$\sum_{k=1}^n (-1)^{k-1} k \binom{n}{k} = 0.$$

4. Find  $\sqrt{1 + \sqrt{-1}}$ .