Math 429 Fall 2005
Assignment 10: Due by Dec 5

1. Show that the characteristic polynomial of a nilpotent linear operator N on n-dimensional vector space V is x^n. (Note, A linear operator N is called *nilpotent* if $N^r = 0$ for some positive integer r. As you know, you can find this statement page 238 as a corollary. But that proof will not be counted. By using the definitions of characteristic polynomial and minimal polynomial, you can get another simply proof.)

2. Find the Jordan form of the matrix

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$

   Note, we may assume base field is $\mathbb{C}$ if you need.

3. Let N_r be an $r \times r$ matrix defined as

$$\begin{pmatrix} 0 & 0 & .. & 0 & 0 \\ 1 & 0 & .. & 0 & 0 \\ 0 & 1 & .. & 0 & 0 \\ .. & .. & .. & .. & .. \\ 0 & 0 & .. & 1 & 0 \end{pmatrix}.$$

   Consider a block diagonal matrix A defined as

$$\begin{pmatrix} N_4 & 0 & 0 & 0 \\ 0 & N_6 & 0 & 0 \\ 0 & 0 & N_9 & 0 \\ 0 & 0 & 0 & N_{13} \end{pmatrix}.$$

   Show A is a nilpotent and find minimal positive integer k such that $A^k = 0$.