

Math 429 Fall 2005
Assignment 2: Due by Sept 19th

1. Consider a set

$$S = \{(a, b) \in \mathbb{R}^2 \mid a \geq 0, b \geq 0\}$$

with the usual summation on \mathbb{R}^2 . Is this a vector space? Justify your answer.

What if

$$S = \{(a, b) \in \mathbb{R}^2 \mid ab \geq 0\}$$

with the above summation?

2. Consider the set of upper triangular matrices defined as

$$N = \left\{ A \in M_{n \times n}(\mathbb{F}) \mid (A)_{ij} = 0 \text{ if } i > j \right\}$$

where \mathbb{F} is a field. Is this a vector space of $M_{n \times n}(\mathbb{F})$? If so, what is the dimension of N ?

3. Let $A \in M_{n \times m}(\mathbb{C})$ and $n \leq m$. Consider a set

$$S = \{X \in M_{m \times 1}(\mathbb{C}) \mid AX = 0 \in M_{n \times 1}(\mathbb{C})\}$$

i.e. set of solutions for $AX = 0$, a homogeneous linear system. Show S is a vector subspace of $M_{m \times 1}(\mathbb{C})$. If the given system is nonhomogeneous, can S be a vector subspace? Justify your answer.

4. Let S be the set of $(x_1, x_2, x_3, x_4, x_5)$ in \mathbb{C}^5 which satisfy

$$\begin{aligned}x_1 + x_2 + x_3 + x_4 + x_5 &= 0, \\3x_1 + 2x_2 + x_3 + x_4 - 3x_5 &= 0, \\x_2 + 2x_3 + 2x_4 + 6x_5 &= 0, \\5x_1 + 4x_2 + 3x_3 + 3x_4 - x_5 &= 0.\end{aligned}$$

Find a basis of S .

5. Let V be a n -dimensional vector space and there are given vector e_1, \dots, e_m . Prove that if $m \geq n + 2$, then there exist numbers a_1, \dots, a_m not all of them equal to zero such that $\sum_{i=1}^m a_i e_i = 0$ and $\sum_{i=1}^m a_i = 0$.