Math 429 Fall 2005
Assignment 5: Due by Oct 17

1. Consider a linear transformation T: $\mathbb{R}^5 \to \mathbb{R}^4$ defined as

$$\begin{aligned} y_1 &= x_1 + x_2 + x_3 + x_4 + x_5, \\ y_2 &= 3x_1 + 2x_2 + x_3 + x_4 - 3x_5, \\ y_3 &= x_2 + 2x_3 + 2x_4 + 6x_5, \\ y_4 &= 5x_1 + 4x_2 + 3x_3 + 3x_4 - x_5 \end{aligned}$$

   where $(x_1, x_2, x_3, x_4, x_5) \in \mathbb{R}^5$ and $(y_1, y_2, y_3, y_4) \in \mathbb{R}^4$. Describe $(\ker(T))^\circ$ and $(\operatorname{Im}(T))^\circ$.

2. Let A be an element in $End\,(n, \mathbb{R})$. For vectors v and w in $\mathbb{R}^n$, we define (v, w) as

$$(v, w) = v_1 \cdot w_1 + ... + v_n \cdot w_n$$
$$\text{where } v = (v_1, .., v_n)^t\,,\ w = (w_1, .., w_n)^t\,.$$

   Show that
$$(v, Aw) = \left(A^t v, w\right).$$

   Hint, If you use the fact $(AB)^t = B^t A^t$ for any $A \in Mat_{n\times m}(\mathbb{R})$ and $B \in Mat_{m\times l}(\mathbb{R})$, your proof is much simpler.