Math 429 Fall 2005
Assignment 6: Due by Oct 24

1. In the following questions, A is an element in $End\,(V,\mathbb{R})$ and a subspace W of V is called A-invariant if $A\,(W) \subset W$.

   (1) Prove or disprove that

$$\ker(A) \subset \ker\left(A^2\right)$$
$$\operatorname{Im}(A) \supset \operatorname{Im}\left(A^2\right).$$

   (2) Show that $A^2 = 0$ if and only if $\operatorname{Im}(A) \subset \ker(A)$.

   (3) Prove or disprove that $\ker(A)$ and $\operatorname{Im}(A)$ are A-invariant.

   (4) Consider a set S_λ defined for some $\lambda \in \mathbb{R}$ as

$$S_\lambda = \{v \in V : Av = \lambda v\}.$$

   Prove or disprove that S_λ is $(A - \lambda I)$-invariant where I is the identity element in $End\,(V,\mathbb{R})$.

   (5) Assume A is invertible, prove or disprove that if W is A-invariant than W is also A^{-1}-invariant.

2. Recall $C(\mathbb{R}) = \{f : \text{continuous function on } \mathbb{R}\}$ is a vector space over $\mathbb{R}$. Consider a function F on $C(\mathbb{R})$ defined as for $f \in C(\mathbb{R})$

$$F(f) = \int_{-\infty}^{\infty} f(x)\,dx.$$

   Is F an element of $(C(\mathbb{R}))^*$, i.e. a linear functional on $C(\mathbb{R})$? Justify your answer.