

Math 429 Fall 2005
Assignment 8: Due by Nov 7

1. Let A be a 2×2 real matrix,

(1) Show A satisfies

$$A^2 - \operatorname{tr}(A)A + \det(A)I_2 = 0$$

where 0 is the 2×2 zero matrix.

(2) For any similar matrix B to A , show

$$B^2 - \operatorname{tr}(A)B + \det(A)I_2 = 0.$$

2. Let A be a 2×2 real matrix defined as

$$\begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}.$$

Consider $A^{2005} = aA + bI_2$. Determine a and b .

3. Let $f(x)$ and $g(x)$ be polynomials in $F[x]$ where F is either \mathbb{R} or \mathbb{C} defined as

$$\begin{aligned} f(x) &= 8x^3 - 1 \\ g(x) &= 2x^4 + x^3 + x^2 + x - 1 \end{aligned}$$

(1) Find the primary decompositions of $f(x)$ and $g(x)$ when F is \mathbb{R} .

(2) Find the primary decompositions of $f(x)$ and $g(x)$ when F is \mathbb{C} .

(3) Find the *g.c.d.* of $f(x)$ and $g(x)$.