

**Math 429   Fall 2005**  
**Assignment 9: Due by Nov 14**

1. Let  $V$  be a finite dimensional vector space over a field  $F$  and  $T$  be a linear operator on  $V$ . Consider eigenvectors  $v_1$  and  $v_2$  of  $T$  with distinct eigenvalues. Show that  $av_1 + bv_2$ , where  $a$  and  $b$  are nonzero scalars, can not be an eigenvector of  $T$ .
2. Let  $T$  be a diagonal operator on  $\mathbb{R}^n$  defined as  $T((x_1, \dots, x_n)) = (a_1x_1, \dots, a_nx_n)$  where  $(x_1, \dots, x_n) \in \mathbb{R}^n$  and  $a_i$ 's are distinct. Describe all the invariant subspaces and show that there are  $2^n$  such spaces.
3. Consider a linear transformation  $A: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  defined as

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}.$$

Find eigenvalues of  $A$  and their eigenspaces.