Searching for Similarities Among Different Things

The title is the base line of my researches on symplectic geometry, differential geometry and mathematical physics. In my earlier works, the geometry of the manifolds with special holonomy was constructed as symplectic geometry and hyperkähler geometry via the vector cross product geometry and mathematical physics. Furthermore, the geometric structures of G_2 and $Spin(7)$-manifolds were studied according to the M-theory. More recently, symplectic Grassmannian spaces and symplectic knot spaces have been defined and investigated by me and N.C. Leung. The geometries on these spaces are tied with symplectic geometry. In my future research plan, I propose to study the relationships between the hierarchy of normed algebras ($\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$ and $\mathbb{O}$) and various geometries. In particular, I would like to construct the special Lagrangian submanifolds in cotangent bundles from isoparametric geometry.

1. Geometry on Manifolds with Special Holonomy

The vector product in $\mathbb{R}^3$ was generalized by Gray ([7],[13]) to the product of tangent vectors, called the *vector cross product* (abbrev. VCP). He listed the Riemannian manifolds with VCP structures on their tangent bundles according to their holonomy groups such as symplectic (or Kähler) manifolds, G_2-manifolds, $Spin\,(7)$-manifolds and oriented manifolds. The first three of these are manifolds with special holonomy, and they play important roles in string theory, M-theory and F-theory, respectively. We can also consider complex analogues of VCP, and it turns out that they are either Calabi-Yau geometry or hyperkähler geometry. These are again corresponding to manifolds with special holonomy, and they play important roles in Mirror Symmetry. Therefore, from a holonomy point of view, VCP gives a coherent description of all possible special holonomy structures on Riemannian manifolds, as classified by Berger. This is the motto of my research in this section.

1.1 Vector Cross Products and Higher Dimensional Knot spaces.[26][27](Joint work with N.C. Leung) On a symplectic manifold, there are interesting questions such as the Gromov-Witten invariants (counting holomorphic curves) and Floer homology (the study of holomorphic curves with boundaries in the intersection of Lagrangian submanifolds). For a manifold N with a symplectic structure ω, Lagrangian submanifolds are the maximal dimensional (half of dimension of N) ones among the submanifolds with ω vanishing, and holomorphic curves are the minimal(in fact, two) dimensional ones whose tangent spaces are preserved by a compatible almost complex structure. In fact, these submanifolds are defined only under the presence of the symplectic tensors, or equivalently of the corresponding almost complex structures. Therefrom, we pointed out these almost complex structures are one type of VCPs, and develop natural analogues of those submanifolds in the context of *VCP*, which are named *instantons* and *branes.*

For a manifold M with a VCP, we study the geometry of *instantons,* which are submanifolds in M preserved by the VCP, equivalently calibrated by the corresponding VCP form. Therefore, instantons are always absolute minimal submanifolds in M. When an instanton is a submanifold with boundary in M, we require its boundary to lie inside a *brane* in order to have a Fredholm theory for the free boundary value problem. In fact, instantons and branes for VCP geometry are supersymmetric cycles in the physical theories corresponding to each VCPs.

Mfd (VCP)	Symplectic Mfds	G_2-mfds	$Spin(7)$-mfds	Oriented Riem. mfds
Instantons	Holomorphic curves	Associative submfds	Cayley submfds	Open submanifolds
Branes	Lagrangian submfds	Coassociative submfds	N/A	Hypersurfaces

We define a tensor τ on M, and we prove that instantons A in M can be characterized by the condition $\tau|_A = 0$. Such a characterization is useful in describing the deformations of instantons. For a brane C in M, its normal bundle is naturally identified with the certain space of VCP forms on C. We prove that such differential forms on C that are closed parameterize the infinitesimal deformations of branes. Furthermore we show they are always unobstructed, namely the moduli spaces of branes are always smooth.

We also introduce the complex analog of VCP called the *complex vector cross product* (abbrev. $\mathbb{C}$-VCP). We prove that there are only two classes of closed Kähler manifolds with $\mathbb{C}$-VCP, which are Calabi-Yau manifolds and hyperkähler manifolds.

For $\mathbb{C}$-VCP, the definition of instantons depends on a parameter called the phase θ. We prove that they can be characterized as the vanishing of certain differential forms on them. Such characterization is needed to describe the deformation of these instantons. Inspired by the Schoen school's studies ,[10],[35],[37], we define two types of branes of $\mathbb{C}$-VCP, *D-branes* and *N-branes* corresponding to the Dirichlet type and Neumann type boundary value problems for instantons, respectively.

Mfd ($\mathbb{C}$-VCP)	Instantons	N-Branes	D-Branes
Calabi-Yau mfds	Special Lagrangian$_{\theta=0}$	Complex Hypersurfaces	Special Lagrangian$_{\theta=-\pi/2}$
hyperkähler mfds	I-holomorphic curves	J-complex Lagrangians	K-complex Lagrangians

Using the Riemannian metric on M, any r -fold closed VCP determines a closed differential form ϕ of degree $r+1$. By transgressing ϕ, we obtain a two form Ω on the (multi-dimensional) knot space $\mathcal{K}_\Sigma M$ of M which is defined as $Map\,(\Sigma, M)\,/Diff\,(\Sigma)$ where Σ is any smooth manifold of dimension $r-1$. The geometry of VCP on M can be realized as the symplectic geometry of the knot space $\mathcal{K}_\Sigma M$. We prove that the knot space $\mathcal{K}_\Sigma M$ has a natural symplectic structure Ω. Furthermore, branes (resp. instantons) in M correspond to Lagrangian submanifolds (resp. holomorphic curves) in $\mathcal{K}_\Sigma M$.

For $\mathbb{C}$-VCP, the geometry of Calabi-Yau n-fold X can be realized as the complex symplectic geometry of the isotropic knot space $\hat{\mathcal{K}}_\Sigma X$ of $(n-2)$-dimensional isotropic knot in X. We construct this isotropic knot space $\hat{\mathcal{K}}_\Sigma X$ as a symplectic quotient of $Map\,(\Sigma, X)$ by $Diff\,(\Sigma)$, and show it has a natural holomorphic symplectic structure. Furthermore, we demonstrate that $\hat{\mathcal{K}}_\Sigma C$ is a J-complex(resp. K-complex) Lagrangian submanifold in $\hat{\mathcal{K}}_\Sigma X$ if and only if C is a complex hypersurface(resp. a special Lagrangian submanifold with phase $-\pi/2$) in X.

Further Research: Instantons in both VCP and $\mathbb{C}$-VCP settings are calibrated submanifolds. This gives a unified way to explain the calibrating property of many such examples, as studied by Harvey and Lawson in [15]. I will continue to study the calibration geometry from this point of view.

Since the geometry of VCP on the manifolds M corresponds to the symplectic geometry of their knot spaces $\mathcal{K}_\Sigma M$, it is natural to study the intersection theory of branes and count the number of instantons bounding them, similar to the Floer's homology theory of Lagrangian intersections. For G_2-manifolds, the problem is counting associative submanifolds bounding nearby coassociative submanifolds. For a very small associative submanifold, this must be approached as a problem in a four manifold since the zero section C in $\wedge_2^+(C)$ is always coassociative for certain G_2-metric on its neighborhood where the bundle $\wedge_2^+(C)$ is topologically trivial(see [8]). When A has a very small volume, this problem is closely related to the Seiberg-Witten invariants of the four dimensional coassociative submanifolds [31]. For general case, the problem is very complicated since we have to explain the bubble phenomenon, which has not been established. I am interested in the coassociative submanifolds appearing in $\wedge_2^+(C)$ in a way similar to counting instantons. Q.S. Chi and I have been investigating characterization and examples of coassociative submanifolds along this line.

The Calabi-Yau geometry on X is related to the holomorphic symplectic geometry of $\hat{\mathcal{K}}_\Sigma X$. This is particularly interesting when X is a Calabi-Yau threefold since $\Sigma \subset X$ is simply a loop and the isotropic condition is automatic for $\hat{\mathcal{K}}_\Sigma X$. The Calabi-Yau threefold appears in the string theory for compactifying a ten-dimensional space-time on it. And $\hat{\mathcal{K}}_\Sigma X$ is the space of loops (or *string*) coupled with flat line bundles in X, up to deformations of strings along their complexified tangent directions. Furthermore, I expect the Strominger-Yau-Zaslow mirror transformation [39] for Calabi-Yau threefold are tied with the twistor rotation for the holomorphic symplectic manifolds $\hat{\mathcal{K}}_\Sigma X$. One of my plans is to verify this relationship for Calabi-Yau threefolds and extend to arbitrary n-folds.

1.2 Geometric Structures of G_2 and *Spin*(7)-manifolds.[25](Joint work with N.C. Leung) A G_2-manifold M is a manifold with a parallel positive three form Ω (in fact, parallel 3-fold VCP form). Because of the natural inclusion of Lie groups $SU(3) \subset G_2$, the product of a Calabi-Yau threefold X with a circle S^1 has a canonical G_2-structure. Therefore, the mirror symmetry conjecture for Calabi-Yau manifolds is better be understood as duality transformations, or equivalently the geometry of G_2-manifolds. In fact, this corresponds to the physical consideration of the *M-theory* on $M \times \mathbb{R}^{3,1}$ with a G_2-manifold M, for instance by Acharya, Atiyah, Vafa, Witten and others [1] [4] [3] [5]. According to this, we study natural geometric structures on various moduli spaces attached to M, as follows.

The geometry of a G_2-manifold is reflected by its calibrated submanifolds; *coassociative* submanifolds and *associative* submanifolds [15]; and Yang-Mills bundles [12]. As in physics [33], we study *supersymmetric cycles* in G_2-manifold, which are calibrated submanifolds together with *deformed Yang-Mills bundles* over them.

Calibrated Submfds	Associative submfds	Coassociative submfds	G_2-manifold
Yang-Mills bundles	Unitary flat	ASD	Deformed Donaldson-Thomas

On moduli spaces of these cycles (denoted as $\mathcal{M}^{coa}(M), \mathcal{M}^{ass}(M)$ and $\mathcal{M}^{bdl}(M)$), we define canonical three forms and four forms which are defined using the Clifford multiplication on spinor bundles and the Lie algebra structure on the space of self-dual two forms. In the flat situation such canonical three forms determines G_2-structures on both $\mathcal{M}^{coa}(M)$ and $\mathcal{M}^{ass}(M)$. In general the moduli space of associative (resp. coassociative) submanifolds can be regarded as a coassociative (resp. associative) subspace of $\mathcal{M}^{ass}(M)$ (resp. $\mathcal{M}^{coa}(M)$).

We introduce natural symmetric cubic tensors and differential forms on these moduli spaces. They are Yukawa couplings and correlation functions in the physics of M-theory. We conjecture the Yukawa coupling and the first Pontryagin class p_1 describe structures of (co-)associative fibrations.

Next we propose a duality transformation on the geometry of G_2-manifolds analogous to the mirror symmetry conjecture by Strominger, Yau and Zaslow [39]. Roughly speaking it should be given by a fiberwise Fourier transformation on a coassociative T^4-fibration on M. We partially verify our proposed conjecture in the flat case, in the spirit of [32][29].

We also discuss similar structures and transformations for *Spin*(7)-manifolds.

Further Research: For a G_2-manifold M with $b_1(M) = 0$, I plan to construct a G_2-reduction as an analogue of symplectic reductions. Symplectic reductions are the methods to produce symplectic manifolds from symplectic(or presymplectic) manifolds. In particular, when a symplectic manifold has a Hamiltonian group action, the corresponding moment map induces a symplectic reduction. In contrast with the symplectic reductions, the G_2-reduction on G_2-manifolds will not produce G_2-manifolds, because G_2-manifolds must be 7-dimensional manifolds and the produced manifolds have the lower dimensions. However, I propose the following two types of G_2-reductions as analogues of symplectic reductions.

(i) Symplectic structures are 1-fold VCPs, and the manifolds with 2-fold VCP are 7-dimensional and 3-dimensional. Therefore, the G_2-reduction will produce 3-dimensional manifolds from G_2-manifolds. In fact, these 3-folds are the spaces of coassociative leaves.

(ii) I consider G_2-Hamiltonian group actions and moment maps. The G_2-reduction is defined as an analogue of hyperkähler reductions and produces a 4-manifold with interesting structures such as a hyperkähler structure.

For the second types of G_2-reductions, the calibrated cycles in a G_2-manifold M are related to the calibrated cycles in the corresponding 4-manifold X. Furthermore, I expect the mirror symmetry on a G_2-manifold M is related to the mirror symmetry on X. This will be another evidence for the conjectures for G_2-manifolds in [25] which N.C. Leung and I proposed.

Similarly to G_2-reductions, I also plan to construct $Spin(7)$-reductions. Moreover, I am looking for $Spin(7)$-manifolds with many topological restrictions, which will play important roles for $Spin(7)$-reductions. According to the works of D.D. Joyce [18][19], G. Jang and I obtained new examples of $Spin(7)$-manifolds from the orbifolds induced from torus[17], and I plan to construct $Spin(7)$-manifolds from Calabi-Yau manifolds.

2. Symplectic Geometry on Symplectic Grassmannians

Symplectic geometry on a symplectic manifold M is characterized by a nondegenerate closed skew-symmetric 2-tensor ω. The most important features characterized by ω are *Lagrangian* submanifolds and *holomorphic curves.* These two types of submanifolds are playing key roles in the study of Mirror symmetry, Gromov-Witten invariants, and Floer homology. In particular, sigma A-model in the mirror symmetry is modeled with Lagrangian submanifolds equipped with flat unitary bundles. On the other hand, there are suggestions (see [22]) that A-model can be modeled with *coisotropic* submanifolds. Since the tangent space at each point of a coisotropic submanifold contains Lagrangian subspaces, it is natural that the geometry of coisotropic submanifolds consists with Lagrangian geometry. I show the coisotropic submanifolds correspond to Lagrangians in symplectic knot spaces, and the similar correspondence in the linear setting is also studied by N.C. Leung and me.

2.1 Symplectic Knot Spaces.[23] I define the *symplectic knot space* $\mathcal{K}^{Sp}(\Sigma, M)$ as the space of symplectically embedded submanifolds from a $2k$-dimensional oriented closed manifold Σ to a symplectic manifold M with a symplectic structure ω. The space $\mathcal{K}^{Sp}(\Sigma, M)$ has a symplectic structure Ω on $\mathcal{K}^{Sp}(\Sigma, M)$ by transgressing $\omega^{k+1}/(k+1)!$. It is also shown that the space $\mathcal{K}^{Sp}(\Sigma, M)$ and the structure Ω can be obtained by the symplectic quotient method. I explain the correspondence between coisotropic submanifolds in M and Lagrangians in the symplectic knot space. I also define an almost complex structure on the symplectic knot space, and study the correspondence between almost complex submanifolds in M and holomorphic curves in the symplectic knot space.

Further Research: When Σ is a Riemann surface, one can consider a subspace of $\mathcal{K}^{Sp}(\Sigma, M)$ that consists of pseudo-holomorphic maps. The geometry and topology of this subspace has been studied related to Gromov-Witten invariants. It is interesting to explore the geometry and topology of $\mathcal{K}^{Sp}(\Sigma, M)$ along the development of the pseudo-holomorphic curves.

I show the correspondence between coisotropic submanifolds in M and Lagrangians in $\mathcal{K}^{Sp}(\Sigma, M)$, and the deformation theory of coisotropic submanifolds is studied in [34] and [36]. Therefrom it is natural to study the relationships between the deformation theory of coisotropic submanifolds and that of the corresponding Lagrangians in $\mathcal{K}^{Sp}(\Sigma, M)$. Furthermore, we also ask similar questions for the intersection theories on coisotropic submanifolds in M and the Lagrangian intersection theory in $\mathcal{K}^{Sp}(\Sigma, M)$(See [14] for another approach).

2.2 Symplectic Grassmannian Spaces.[28] (Joint work with N.C. Leung) On a $2n$-dimensional real vector space V, the set of all $2k$-dimensional linear subspaces $S \subset V$ is the real Grassmannian $Gr(2k, 2n)$. When V is equipped with a linear complex structure, the complex Grassmannian $Gr_{\mathbb{C}}(k, n)$

parametrizes those subspaces S which are complex. Furthermore it carries a natural complex structure, which plays a very important role in complex geometry. Similarly, we assume V is equipped with a linear symplectic structure ω and define the *Symplectic Grassmannian* $Gr^{Sp}(2k, 2n)$ to be the open subset of $Gr(2k, 2n)$, which consists of symplectic subspaces. We show it has a natural almost symplectic structure induced from ω^{k+1} and also has a natural strong deformation retract to the complex Grassmannian $Gr_{\mathbb{C}}(k, n)$.

We study the symplectic geometry of $Gr^{Sp}(2k, 2n)$. We construct natural Lagrangian cycles $\mathcal{L}_C(2k, 2n)$ in $Gr^{Sp}(2k, 2n)$ from coisotropic(or isotropic) subspaces C in V. These are analogous to the Schubert cells in $Gr_{\mathbb{C}}(k, n)$. We also construct a Lagrangian fibration on $Gr^{Sp}(2k, 2n)$. Furthermore, holomorphic curves in $Gr^{Sp}(2k, 2n)$ can be constructed from higher dimensional complex subspaces in V.

Further Research: We construct natural Lagrangian cycles in $Gr^{Sp}(2k, 2n)$ from coisotropic (or isotropic) subspaces in V. These are analogous to the Schubert cells in the Complex Grassmannian $Gr_{\mathbb{C}}(k, n)$. Recall the Schubert calculus $Gr_{\mathbb{C}}(k, n)$ deals with the intersection theory of its natural complex submanifolds, called the Schubert varieties. N.C. Leung and I plan to develop the *Lagrangian Schubert calculus* for $Gr^{Sp}(2k, 2n)$ in parallel with (or *mirror* with) the usual Schubert calculus for $Gr_{\mathbb{C}}(k, n)$.

We construct a Lagrangian fibration on $Gr^{Sp}(2k, 2n)$ from an isotropic fibration on the symplectic vector space V. This fibration is induced from the orthogonal projection π from V to a $(n+k)$-dimensional coisotropic subspace C. Each generic points in $Gr^{Sp}(2k, 2n)$ corresponds to point in the sub-Grassmannian space $\mathcal{L}_C(2k, 2n)$ that gives a Lagrangian section in $Gr^{Sp}(2k, 2n)$. Non-generic points in $Gr^{Sp}(2k, 2n)$ are characterized as $\dim \pi(S) < 2k$, and related to singular Lagrangian fibers. I plan to study these singular Lagrangian fibers.

3. Geometry and Normed Algebras

One of my research interests is placing various geometries into the hierarchy of *normed algebra*s, real $\mathbb{R}$, complex $\mathbb{C}$, quaternions $\mathbb{H}$ and octonions $\mathbb{O}$. This approach has been appreciated in many aspects of geometry such as [30] and [6]. In particular, Atiyah and Berndt[2] explain the geometry of projective planes($\mathbb{RP}^2$, $\mathbb{CP}^2$, $\mathbb{HP}^2$ and $\mathbb{OP}^2$) for each normed algebras. Note these projective planes appear in the list of cotangent spaces with Ricci flat metric by Stenzel[38] and the list of *Cartan hypersurfaces*; minimal isoparametric hypersurfaces with three distinct principal curvatures; in spheres. Therefrom I consider the followings.

3.1 Special Lagrangian Submanifolds in Cotangent Bundles.[24] A Calabi-Yau manifold is a Kähler manifold with a parallel holomorphic volume form Ω, and a submanifold calibrated by $\operatorname{Re}\Omega$ is called a *special Lagrangian* submanifold. Special Lagrangians have been studied as a type of calibrated submanifolds [15] and recently these attract a lot of attentions thanks to their roles in mirror symmetry on Calabi-Yau manifolds.

For a submanifold X of a manifold M, its conormal bundle N^*X is a Lagrangian submanifold of the cotangent bundle T^*M with the canonical symplectic structure. In particular, Harvey and Lawson showed if M is an Euclidean space $\mathbb{R}^n$, N^*X is a special Lagrangian in $T^*\mathbb{R}^n$ if and only if X is *austere*, namely the shape operator of X in M for each normal vector has the set of eigenvalues invariant under multiplication of -1.

On the other hand, Stenzel[38] showed that the cotangent bundle of a sphere has a complete Ricci-flat metric, i.e. it is a Calabi-Yau manifold. Therefore a natural question is whether the austere submanifolds X in the sphere S^n correspond to special Lagrangians N^*X in T^*S^n. However, one issue is that the Kähler form ω_{sz} corresponding to the Ricci flat metric is not the canonical one. In this regard, I show that conormal bundle N^*X is also a Lagrangian submanifold in T^*S^n for ω_{sz}, and moreover, it is a special Lagrangian if and only if X is austere in S^n[24]. This result is also achieved in [21] independently.

For the examples of austere submanifolds in spheres, I point that minimal isoparametric hypersurfaces and their focal submanifolds in spheres are austere. The *isoparametric hypersurfaces* in a sphere are hypersurfaces with constant principal curvatures. These austere examples are independently observed in [9] and [20] for the construction of minimal Legendrian submanifolds in S^{2n+1} and for the generalization, respectively.

Further Research: Note that Stenzel's result[38] contains cotangent bundles of projective spaces for each normed algebras. I plan to verify that the austere submanifolds of each projective spaces are related the special Lagrangians conormal bundles in the corresponding cotangent bundles. On the other hand, the cotangent bundle $T^*\mathbb{CP}^n$ is an example of noncompact manifolds with the hyperkähler structures (Calabi[11] and Hitchin[16]). The only compact Lagrangian submanifold in $T^*\mathbb{CP}^n$ is the zero section $\mathbb{CP}^n$. And for each submanifold X in $\mathbb{CP}^n$, the conormal bundles of X in $\mathbb{CP}^n$ is Lagrangian in $T^*\mathbb{CP}^n$. Furthermore, it is an interesting question whether austere submanifolds in $\mathbb{CP}^n$ correspond to special Lagrangian conormal bundles in $T^*\mathbb{CP}^n$.

Cartan hypersurfaces are minimal isoparametric hypersurfaces in spheres with three distinct principal curvatures, which are identified with the unit normal bundle of the projective planes ($\mathbb{RP}^2$, $\mathbb{CP}^2$, $\mathbb{HP}^2$ and $\mathbb{OP}^2$) in spheres. Accordingly, I would like to bring the results in [2] in conjunction with the orbit description on these projective planes, which coincides with Cartan hypersurfaces and focal sets along the values of the isoparametric functions to spheres.

References

[1] B.S. Acharya, On Mirror Symmetry for Manifolds of Exceptional Holonomy, Nucl.Phys. B524 269 (1998), no. 1-2, 269–282.

[2] M. Atiyah and J. Berndt, Projective planes, Severi varieties and spheres. Surveys in differential geometry, Vol. VIII (Boston, MA, 2002), 1–27, Surv. Differ. Geom., VIII, Int. Press, Somerville, MA, 2003.

[3] M. Atiyah, J. Maldacena, C. Vafa, An M-theory flop as a large N duality, Strings, branes, and M-theory. J. Math. Phys. 42 (2001), no. 7, 3209–3220.

[4] M. Aganagic, C. Vafa, Mirror symmetry and a G_2 flop, J. High Energy Phys. 2003, no. 5. 061, 16 pp.

[5] M. Atiyah, E. Witten, M-Theory Dynamics On A Manifold Of G_2 Holonomy, Adv. Theor. Math. Phys. 6 (2002), no. 1, 1–106.

[6] John C. Baez, The Octonions, Bull. AMS. 39 no.2. (2001) 145-205.

[7] R.B. Brown and A.Gray, Vector Cross Prodcuts, Comment. Math. Helv. 42 (1967), 222-236.

[8] R. Bryant, Calibrated Embeddings in the Special Lagrangian and Coassociative Cases, Ann. Global Anal. Geom. 18, no. 3-4, 405-435

[9] V. Borrelli and C. Gorodski, Minimal Legendrian submanifolds of $S^{2n+1}$and absolutely area-minimizing cones. Differential Geom. Appl. 21 (2004), no. 3, 337–347.

[10] A. Butscher, Deformations of minimal Lagrangian submanifolds with boundary. Proc. Amer. Math. Soc. 131 (2003), no. 6, 1953–1964.

[11] E. Calabi, Mériques Kähleriennes et fibrés holomorphes, Ann. Ec. Norm. Sup. 12 (1979), 269-294.

[12] S. K. Donaldson and R. P. Thomas, Gauge Theory in Higher Dimensions, The Geometric Universe (Oxford, 1996), Oxford Univ. Press, Oxford (1998) pp. 31-47.

[13] A. Gray, Vector Cross Products on Manifolds Trans. Amer. Math. Soc. 141 (1969) 465–504.

[14] V. L. Ginzburg, Coisotropic Intersections [math.SG/0605186]

[15] R. Harvey, B. Lawson, Calibrated geometries, Acta Math. 148 (1982), 47-157.

[16] N. Hitchin, Hyperkahler manifolds, seminaire Bourbaki, Asterisque 206(1992) 137-166.

[17] G. Jang and J.H. Lee, New examples of compact 8-manifolds with holonomy $Spin(7)$, In preparation.

[18] D.D. Joyce, Compact 8-manifolds with holonomy $Spin(7)$, Inventiones mathematicae 123 (1996), 507-552.

[19] D.D. Joyce, A new construction of compact 8-manifolds with holonomy Spin(7) J.Diff.Geom. 53 (1999) 89-130.

[20] M. Kimura, A generalization of Cartan hypersurfaces. Proceedings of the Ninth International Workshop on Differential Geometry, 51–59, Kyungpook Nat. Univ., Taegu, 2005.

[21] S. Karigiannis and M. Min-Oo, Calibrated subbundles in noncompact manifolds of special holonomy. Ann. Global Anal. Geom. 28 (2005), no. 4, 371–394.

[22] A. Kapustin and D. Orlov, Remarks on A-branes, mirror symmetry, and the Fukaya category. J. Geom. Phys. 48 (2003), no. 1, 84–99.

[23] J.H. Lee, Symplectic Geometry on Symplectic Knot Spaces. Preprint.

[24] J.H. Lee, Special Lagragnians Submanifolds in Cotangent Bundles of Spheres. Preprint.

[25] J.H. Lee and N.C. Leung, Geometric Structures on $G_2$and $Spin(7)$-manifolds, [Math.DG/0202045].

[26] J.H. Lee and N.C. Leung, Instantons and Branes in Manifold with Vector Cross Product, Preprint.

[27] J.H. Lee and N.C. Leung, Higher Dimensional Knot Spaces for Manifolds with Vector Cross Product. Preprint.

[28] J.H. Lee and N.C. Leung, Symplectic Geometry on Symplectic Grassmannian Spaces. Preprint.

[29] N.C. Leung, Mirror symmetry without corrections, Comm. Anal. Geom. 13 (2005), no. 2, 287–331.

[30] N.C. Leung, Riemannian geometry over different normed division algebras. J. Differential Geom. 61 (2002), no. 2, 289–333.

[31] N.C. Leung and X.W. Wang, Intersection theory of coassociative submanifolds in G_2-manifolds and Seiberg-Witten invariants, [Math.DG/0401419].

[32] N.C. Leung, S.Y. Yau, E. Zaslow, From special Lagrangian to Hermitian-Yang-Mills via Fourier-Mukai transform, to appear in Adv. Thero. Math. Phys.. [math.DG/ 0005118].

[33] M. Marino, R. Minasian, G. Moore, and A. Strominger, Nonlinear Instantons from supersymmetric p-Branes, [hep-th/9911206].

[34] Y.G. Oh and J.S. Park, Deformations of coisotropic submanifolds and strong homotopy Lie algebroids. Invent. Math. 161 (2005), no. 2, 287–360.

[35] W.Y. Qiu, Special Lagrangian submanifolds with boundary and Lagrangian free boundary problem, preprint 2003.

[36] W.D. Ruan, Deformation of integral coisotropic submanifolds in symplectic manifolds. J. Symplectic Geom. 3 (2005), no. 2, 161–169.

[37] R. Schoen, Special Lagrangian Submanifolds. Lecture notes by W.Y. Qiu. (2002).

[38] M.B. Stenzel, Ricci-flat metrics on the complexification of a compact rank one symmetric space. Manuscripta Math. 80 (1993), no. 2, 151–163.

[39] A. Strominger, S.-T. Yau, and E. Zaslow, Mirror Symmetry is T-Duality, Nuclear Physics B479 (1996) 243-259.