Homework 2: Due 02/06/2018

1. (10 points) Problem 19 on page 302 of Shao (2003).
   Using the empirical Bayes method and the method of moments to derive the priors under squared error loss function. And plug the estimated priors to get empirical Bayes decisions in (b)-(d) for Q1 in HW1.

2. (10 points) Problem 21 on page 302 of Shao (2003).

3. (10 points) Problem 25 on page 303 of Shao (2003).

4. (10 points) Please show that the variance and risk of any location invariant estimator are constants (independent of location parameter).

5. (10 points) Suppose a location invariant estimator δ_0 with finite risk is independent of the ancillary statistics $D = (X_1 - X_n, X_2 - X_n, \cdots, X_{n-1} - X_n)^T$.

   (a) Prove that the MRIE is $\delta^*(X) = \delta_0(X) - u^*$, where
   $$u^* = \arg\min_u E_0[L(\delta_0(X) - u)],$$
   which is a constant and independent of the value of D.
   Remark: if δ_0 is sufficient and complete statistics , then by Basu's theorem, $\delta_0 \perp\!\!\!\perp D$.

   (b) If, in addition, the distribution of δ_0 is symmetric about the location parameter θ, and the loss function L is convex and even, then $u^* = 0$ and δ_0 is an MRIE.

   (c) As an example of above results, show that, in exponential distributions $E(\theta, 1)$, the MRIE for θ is $X_{(1)} - \log 2/n$ under absolute loss.