

# Math 141: Accelerated Calculus I

6 (a)  $\lim_{x \rightarrow -2^-} g(x) = -1$       (b)  $\lim_{x \rightarrow -2^+} g(x) = 1$

(c) Since the left (a) and right (b) limits are different,  $\lim_{x \rightarrow -2} g(x)$  does not exist.

(d)  $g(-2) = 1$

(e)  $\lim_{x \rightarrow 2^-} g(x) = 1$       (f)  $\lim_{x \rightarrow 2^+} g(x) = 2$ .

(g) Since the left and right limits are different,  $\lim_{x \rightarrow 2} g(x)$  does not exist.

(h)  $g(2) = 2$

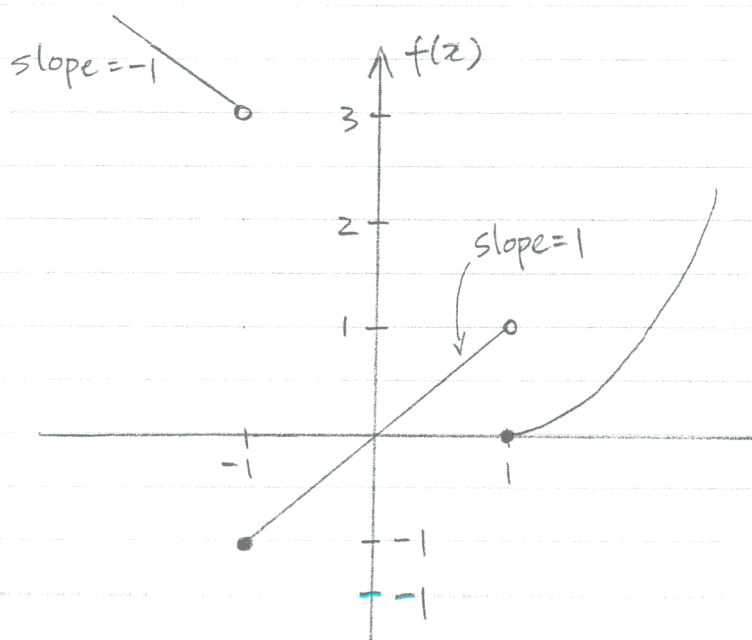
(i)  $\lim_{x \rightarrow 4^+} g(x)$  does not exist, since the values of  $g(x)$  oscillate infinitely between the values 1.5 and 2 as  $x$  approaches 4 from the right.

(j)  $\lim_{x \rightarrow 4^-} g(x) = 2$

(k)  $g(0)$  does not exist. (The function is undefined at  $x=0$ )

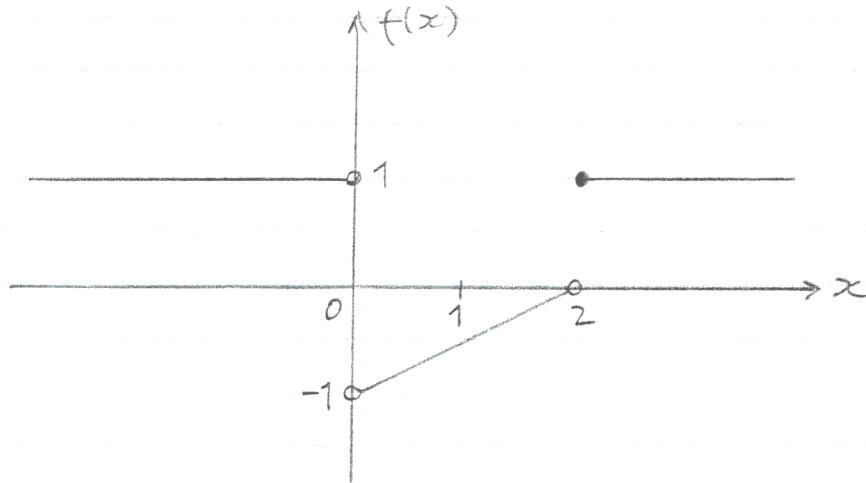
(l)  $\lim_{x \rightarrow 0} g(x) = 0$ . (Since  $\lim_{x \rightarrow 0^-} g(x) = \lim_{x \rightarrow 0^+} g(x) = 0$ )

12.



$\lim_{x \rightarrow a} f(x)$  exists for every real  $a$ , except for  $-1$  and  $1$

14.



38. As  $v \rightarrow c^-$ ,  $v^2 \rightarrow (c^2)^-$   
 $\Rightarrow \frac{v^2}{c^2} \rightarrow 1^-$   
 $\Rightarrow 1 - \frac{v^2}{c^2} \rightarrow 0^+$   
 $\Rightarrow \sqrt{1 - \frac{v^2}{c^2}} \rightarrow 0^+$   
 $\Rightarrow \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \rightarrow \infty$   
 $\Rightarrow m \rightarrow \infty$

Section 2.4.

26. Let  $\delta = \sqrt[3]{\epsilon}$   
 $\therefore 0 < |x| < \delta \Rightarrow |x|^3 < \delta^3 = \epsilon$   
 $\Rightarrow |x^3 - 0| < \epsilon$   
 $\therefore \lim_{x \rightarrow 0} x^3 = 0$ .

30.  $0 < |x-a| < \delta \Rightarrow |f(x) - L| < \epsilon$   
 $0 < |x-3| < \delta \Rightarrow |x^2 + x - 4 - 8| < \epsilon$   
 $\Rightarrow |x-3||x+4| < \epsilon$   
 $|x+4| < c \Rightarrow |x-3||x+4| < c|x-3|$   
 $c|x-3| < \epsilon \Rightarrow |x-3| < \frac{\epsilon}{c} = \delta$   
 $\therefore |x-3| < 1, 2 < x < 4$   
 $\Rightarrow 6 < x+4 < 8$   
 $\Rightarrow c = 8$ .

Let  $\delta = \min \left\{ 1, \frac{\epsilon}{8} \right\}$   
 $|x^2 + x - 12| = |x+4||x-3| < 8 \cdot \frac{\epsilon}{8} = \epsilon$ .