Sec. 11.2

(10) (a) BOTH $\sum_{i=1}^{n} a_i$ & $\sum_{j=1}^{n} a_j$ mean the sum of first n terms of the sequence $\{a_n\}$, i.e., the n^{th} partial sum

(b) $\sum_{i=1}^{n} a_j = \underbrace{a_j + a_j + \cdots + a_j}_{n \text{ terms}} = n a_j$ NOT the same as $\sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n$

(20) $\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}} = 3 \sum_{n=1}^{\infty} \left(\frac{e}{3}\right)^n$, first term $3\left(\frac{e}{3}\right)^1 = e$, ratio $r = \frac{e}{3}$, $\therefore$ GEOMETRIC series

$\because |r| < 1$, CONVERGE, $S = \frac{e}{1 - \frac{e}{3}} = \frac{3e}{3-e}$

(24) $\because \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^2+2n+1}{n^2+2n} = \lim_{n\to\infty} \left(1 + \frac{1}{n^2+2n}\right) = 1 \neq 0$

$\therefore$ By Test for Divergence, $\sum_{n=1}^{\infty} \frac{(n+1)^2}{n(n+2)}$ DIVERGES

(28) i.e, Difference of 2 con geometric series

$\sum_{n=1}^{\infty}[(0.8)^{n-1} - (0.3)^n] = \sum_{n=1}^{\infty}(0.8)^{n-1} - \sum_{n=1}^{\infty}(0.3)^n = \frac{1}{1-0.8} - \frac{0.3}{1-0.3} = 5 - \frac{3}{7} = \frac{32}{7}$

(36) $0.\overline{73} = \frac{73}{10^2} + \frac{73}{10^4} + \cdots = \frac{73/10^2}{1 - 1/10^2} = \frac{73/100}{99/100} = \frac{73}{99}$

(42) $\sum_{n=1}^{\infty}(x-4)^n$ Geometric w/ $r = x-4$

$\therefore$ series conv $\Leftrightarrow |r| < 1 \Leftrightarrow |x-4| < 1 \Leftrightarrow 3 < x < 5$

then sum of series $S = \frac{x-4}{1-(x-4)} = \frac{x-4}{5-x}$

Sec. 11.3

(4) $f(x) = 1/\sqrt[4]{x} = x^{-1/4}$ continuous, positive, & $\downarrow$ on $[1, \infty)$

$\therefore$ we can apply Integral Test

$\int_1^{\infty} x^{-1/4}\,dx = \lim_{t\to\infty} \int_1^t x^{-1/4}\,dx = \lim_{t\to\infty} \left[\frac{4}{3}x^{3/4}\right]_1^t = \lim_{t\to\infty}\left(\frac{4}{3}t^{3/4} - \frac{4}{3}\right) = \infty$

$\therefore \sum_{n=1}^{\infty} f(n)$ DIVERGES

(18) $\because f(x) = \frac{1}{x^2-4x+5} = \frac{1}{(x-2)^2+1}$ CONTINUOUS, POSITIVE & $\downarrow$ on $[2, \infty)$

$\therefore$ we can apply Integral Test

$$\int_2^\infty f(x)\,dx = \lim_{t\to\infty}\int_2^t f(x)\,dx = \lim_{t\to\infty}\int_2^t \frac{1}{(x-2)^2+1}\,dx = \lim_{t\to\infty}\left[\tan^{-1}(x-2)\right]\Big|_2^t$$

$$= \lim_{t\to\infty}\left[\tan^{-1}(t-2) - \tan^{-1}0\right] = \frac{\pi}{2}$$ $\therefore$ series CONVERGES.

Also means, $\sum_{n=1}^\infty \frac{1}{n^2-4n+5}$ CONVERGES.

(34) $\because f(x) = \frac{1}{x(\ln x)^2}$ POSITIVE & CONTINUOUS

$\because f'(x) = -\frac{\ln x+2}{x^2(\ln x)^3}$ is NEGATIVE for $x > 1$

$\therefore$ we can apply Integral Test

using (2), condition: $0.01 > \int_n^\infty \frac{dx}{x(\ln x)^2} = \lim_{t\to\infty}\left[\frac{-1}{\ln x}\right]_n^t = \frac{1}{\ln n}$

$0.01 > \frac{1}{\ln n} \Rightarrow n > e^{100}$

$\therefore$ we need e^{100} terms, but this is PROBLEMATIC $\because e^{100} \approx 2.7\times10^{43}$

Sec 11.4

(16) $\frac{1}{\sqrt{n^3+1}} < \frac{1}{\sqrt{n^3}}$, $\therefore \sum_{n=1}^\infty \frac{1}{\sqrt{n^3+1}}$ CONV. by comparison w/ CONV. p-series $\sum_{n=1}^\infty \frac{1}{\sqrt{n^3}} = \sum_{n=1}^\infty \frac{1}{n^{3/2}}$ $\left(p = \frac{3}{2} > 1\right)$

(20) for $a_n = \frac{1+2^n}{1+3^n}$ & $b_n = \frac{2^n}{3^n}$, use Limit Comparison Test

$$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{(1/2)^n+1}{(1/3)^n+1} = 1 > 0$$

$\because \sum_{n=1}^\infty b_n$ CONV. (geom. series w/ $|r| = \frac{2}{3} < 1$)

$\therefore \sum_{n=1}^\infty \frac{1+2^n}{1+3^n}$ also CONV.