Sec. 13.4

(12) given $r(t) = \langle t^2, \ln t, t \rangle$

4 pt

$v(t) = r'(t) = \langle 2t, \frac{1}{t}, 1 \rangle$

$a(t) = v'(t) = \langle 2, -\frac{1}{t^2}, 0 \rangle$

$|v(t)| = \sqrt{(2t)^2 + (\frac{1}{t})^2 + 1^2} = \sqrt{4t^2 + t^{-2} + 1}$

(32) $\because r(t) = \langle 1+t, t^2 - 2t, 0 \rangle$

$\therefore r'(t) = \langle 1, 2t-2, 0 \rangle$; $|r'(t)| = \sqrt{1^2 + (2t-2)^2} = \sqrt{1 + 4t^2 - 8t + 4} = \sqrt{4t^2 - 8t + 5}$

14 pt

$r''(t) = \langle 0, 2, 0 \rangle$

$$r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ 1 & 2t-2 & 0 \\ 0 & 2 & 0 \end{vmatrix} = 2k$$

$$\therefore a_T = \frac{r'(t) \cdot r''(t)}{|r'(t)|} = \frac{2(2t-2)}{\sqrt{4t^2 - 8t + 5}}$$

$$a_N = \frac{|r'(t) \times r''(t)|}{|r'(t)|} = \frac{2}{\sqrt{4t^2 - 8t + 5}}$$

Sec. 14.1

(30) (a) VI. $\because$ trace in $x=0$ is $z = |y|$ & in $y=0$ is $z = |x|$

5 pt each) ×6 = 30 pt

(b) V $\because$ trace in $x=0$ is $z=0$, & in $y=0$ is $z=0$

(c) I. $\because$ trace in $x=0$ is $z = \frac{1}{1+y^2}$ & in $y=0$ is $z = \frac{1}{1+x^2}$

& $f \to 0$ for large x & y.

(d). IV. $\because$ trace in $x=0$. is $z = y^4$ & in $y=0$ is $z = x^4$.
in $z=0$ is $0 = (x^2 - y^2)^2$ i.e., $y = \pm x$

(e) II. $\because$ trace in $x=0$ is $z = y^2$. & in $y=0$ is $z = x^2$
in $z=0$ is $0 = (x-y)^2$ i.e., $y = x$

(f) III. $\because$ trace in $x=0$ is $z = \sin|y|$. & in $y=0$ is $z = \sin|x|$
note that the graph is oscillating $\Rightarrow$ relates to trigonometric fcns.

Sec 14.2

(8) $f(x,y) = \dfrac{x^2+\sin^2 y}{2x^2+y^2}$

IF approach (0,0) along x-axis, (i.e, $y=0$)

$f(x,0) = \dfrac{x^2}{2x^2} = \frac{1}{2}$ for $x \neq 0$ $\therefore f(x,y) \to \frac{1}{2}$ #

14 pt IF approach (0,0) along y-axis, (i.e., $x=0$)

$f(0,y) = \dfrac{\sin^2 y}{y^2} = \left(\dfrac{\sin y}{y}\right)^2$

note $\lim\limits_{y\to 0} \dfrac{\sin y}{y} = 1$ $\therefore f(x,y) \to 1$ #

$\because$ f has 2 different limits along two different approaches.
Limit does NOT exist.

(14) use Squeeze Thm. to show $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2\sin^2 y}{x^2+2y^2} = 0$

$0 \le \dfrac{x^2\sin^2 y}{x^2+2y^2} \le \sin^2 y$ $\because \dfrac{x^2}{x^2+2y^2} \le 1$

14pt & $\sin^2 y \to 0$ as (x,y) approach $(0,0)$

$\therefore \lim\limits_{(x,y)\to(0,0)} \dfrac{x^2\sin^2 y}{x^2+2y^2} = 0$

(34) $f(x,y,z) = \sqrt{x+y+z} = h(g(x,y,z))$ where $g(x,y,z) = x+y+z$ continues everywhere

$h(t) = \sqrt{t}$ is CONTINUOUS on its domain $\{t \mid t \ge 0\}$

14pt $\therefore$ f is continuous on its domain $\{(x,y,z) \mid x+y+z \ge 0\}$

i.e., f is CONTINUOUS ON & ABOVE the plane $z = -x-y$.