Math 142 – Spring 2004 Name:

Quiz 5

(1) Find an equation of the tangent plane to the given surface at the specified point.

$$z = 4x^2 - y^2 + 2y, \quad (-1, 2, 4).$$

$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$

$f_x(x, y) = 8x \quad f_x(-1, 2) = -8 \quad f_y(x, y) = 2 - 2y \quad f_y(-1, 2) = -2$

$z - 4 = -8(x + 1) - 2(y - 2)$

$8x + 2y + z - 4 + 8 - 4 = 0$

$$\boxed{8x + 2y + z = 0}$$

(2) Find the indicated partial derivatives.

$$R = \ln(u^2 + v^2 + w^2), \quad u = x + 2y, \quad v = 2x - y, \quad w = 2xy; \quad \frac{\partial R}{\partial x}, \frac{\partial R}{\partial y}$$

when $x = y = 1$.

$\frac{\partial R}{\partial x} = \frac{\partial R}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial R}{\partial v}\frac{\partial v}{\partial x} + \frac{\partial R}{\partial w}\frac{\partial w}{\partial x} = \frac{2u \cdot 1 + 2v \cdot 2 + 2w \cdot 2y}{u^2 + v^2 + w^2}$

$x = y = 1 \Rightarrow u = 3, v = 1, w = 2.$

$\left.\frac{\partial R}{\partial x}\right|_{x=y=1} = \frac{6 + 4 + 8}{9 + 1 + 4} = \frac{18}{14} = \frac{9}{7}$

$\frac{\partial R}{\partial y} = \frac{\partial R}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial R}{\partial v}\frac{\partial v}{\partial y} + \frac{\partial R}{\partial w}\frac{\partial w}{\partial y} = \frac{2u \cdot 2 + 2v \cdot (-1) + 2w \cdot 2x}{u^2 + v^2 + w^2}$

$\left.\frac{\partial R}{\partial y}\right|_{x=y=1} = \frac{12 - 2 + 8}{14} = \frac{9}{7}$