(1) (a) Show that a space $X$ is compact if and only if the map $f : X \rightarrow \{\text{point}\}$ is proper.
(b) Prove the converse of the theorem proved in class: A continuous map $f : X \rightarrow Y$ is proper if for any $Z$, the map $f \times \text{Id} : X \times Z \rightarrow Y \times Z$ is closed. (Hint: Use $Z$ to be the one point compactification of $X$)
(c) We say that an infinite sequence $\{x_i\}$ with $x_i \in X$ escapes to infinity any compact subset of $X$ contains only finitely many of the $x_i$'s. Show that a continuous map $f : \mathbb{R}^n \rightarrow Y$ is proper if and only if for any infinite sequence $\{x_i\} \subset \mathbb{R}^n$ escaping to infinity, the sequence $\{f(x_i)\} \subset Y$ is infinite and escapes to infinity.

(2) Show that $\mathbb{Q}$ with the usual topology is not locally compact.

(3) Let $X$ be the set of continuous functions on the closed interval endowed with the supremum metric.
(a) Show that $X$ is locally compact if and only if the closed ball $B(0, c) = \{f \in X | d(f, 0) \leq c\}$ for any $c > 0$ where of course $d(f, 0) = \sup\{f(x) | x \in [0, 1]\}$ is compact.
(b) Show that the sequence of functions, $f_n(x) = x^n$ have no limit in $X$.
(c) Deduce that $X$ is not locally compact.