Math 418, Homework 7, due March 29th 2011

(2) Do problem 7, page 335.
(3) Let \( f(z) \) be a polynomial with complex coefficients of degree at least two.
   
   (a) Show that the map \( \phi : \mathbb{C} \to \mathbb{C} \) given by \( a \mapsto f(a) \) is never a covering map. (Hint: see the next part).

   (b) Let \( R \subset \mathbb{C} \) be the finite set of points where \( f'(z) = 0 \) (called the ramification locus) and let \( B = \phi(R) \) (called the branch locus). Show that the map \( \phi : \mathbb{C} - \phi^{-1}(B) \to \mathbb{C} - B \) is a covering map.

(4) We call a covering \( p : E \to B \) finite, if \( p^{-1}(b) \) is finite for all \( b \in B \).
   
   (a) If \( p : E \to B \) is finite and \( B \) is connected, show that the cardinality of \( p^{-1}(b) \) is constant, independent of \( b \in B \).

   (b) If \( p : E \to B \) is finite and \( B \) is compact, show that \( E \) is compact.