Math 418, Homework 7, due March 29th 2011

(1) Do problem 3, page 330.

(2) Do problem 7, page 335.

(3) Let $f(z)$ be a polynomial with complex coefficients of degree at least two.

  (a) Show that the map $\phi : \mathbb{C} \to \mathbb{C}$ given by $a \mapsto f(a)$ is never a covering map. (Hint: see the next part).

  (b) Let $R \subset \mathbb{C}$ be the finite set of points where $f'(z) = 0$ (called the ramification locus) and let $B = \phi(R)$ (called the branch locus). Show that the map $\phi : \mathbb{C} - \phi^{-1}(B) \to \mathbb{C} - B$ is a covering map.

(4) We call a covering $p : E \to B$ finite, if $p^{-1}(b)$ is finite for all $b \in B$.

  (a) If $p : E \to B$ is finite and B is connected, show that the cardinality of $p^{-1}(b)$ is constant, independent of $b \in B$.

  (b) If $p : E \to B$ is finite and B is compact, show that E is comapct.