

## HOMEWORK 11, DUE THU APR 22ND

*All solutions should be with proofs, you may quote from the book or from previous home works*

- (1) Find the degrees of the splitting fields over  $\mathbb{Q}$  for the following polynomials.
  - (a)  $X^4 + 1$ .
  - (b)  $X^6 + X^3 + 1$ .
- (2) If  $p$  is a prime number, show that the splitting field of  $X^p - 1$  over  $\mathbb{Q}$  has degree  $p - 1$ .
- (3) Let  $P(X) = X^3 + aX + b$ ,  $a, b \in \mathbb{Q}$  and let  $K$  its splitting field over  $\mathbb{Q}$ . Find all possible degrees of  $K$  over the rationals.
- (4) Let  $\phi : \mathbb{Q}(2^{1/3}) \rightarrow \mathbb{Q}(2^{1/3})$  be an automorphism. Show that  $\phi$  is the identity.
- (5) Let  $\phi : \mathbb{R} \rightarrow \mathbb{R}$  be a field automorphism. Show that  $\phi$  is the identity. (Hint: Show that if  $a < b$ ,  $\phi(a) < \phi(b)$ .)