1. Let $G$ be a finite group and $n$ a positive integer relatively prime to the order of $G$. Then prove that the map $G \to G$ given by $x \mapsto x^n$ is bijective.

2. Prove that if $R$ is a Unique factorisation domain, then so is $R[X]$.

3. Given two odd integers $a, b$, show that you can find an integer $n$ such that $n \equiv a \mod 34$ and $n \equiv b \mod 54$.

4. Show that the polynomial $X^p - X + 1$ is irreducible over $\mathbb{F}_p$.

5. Let $S \subset \mathbb{Z}$ be a multiplicatively closed subset and let $M = \mathbb{Q}/S^{-1}\mathbb{Z}$. Find necessary and sufficient conditions on $S$ so that $\text{Ass } M$ is finite.

6. Give an example with proof of two non-zero modules $M, N$ over a commutative ring with 1 so that $M \otimes N = 0$.

7. Compute the character for the standard representation of $S_4$ over $\mathbb{C}$ and prove that it is faithful.