Problem Set 2

due Wednesday Sept. 30

(1) (a) Prove Proposition I.B.4.1; and (b) verify the first two bullets of Example I.B.4.1.
(2) Prove the " $\Longrightarrow$ " direction of Proposition I.B.4.5 (which should really be Prop. 4, not 5...).
(3) (a) Referring to the construction on p. 7 of I.B.4, determine the projections π for Sp_4 which induce Hodge numbers $(2,2)$ and $(1,1,1,1)$ [weights 1 and 3 resp.] on the standard (4-diml) representation V. [Note: the roots may be visualized as a three-by-three grid of dots,[1] with the center "dot" representing the 2-diml Cartan subalgebra $\mathfrak{t}$, and the other root-spaces 1-dimensional. Usually the root at $(1,-1)$ is called σ_1, and that at $(0,1)$ is called σ_2; the others are $\sigma_1+\sigma_2$, $\sigma_1+2\sigma_2$, and the negatives of all these. The compact roots are $\pm\sigma_2$. The weights of V are $\pm\frac{1}{2}\sigma_1$ and $\pm(\frac{1}{2}\sigma_1+\sigma_2)$.]
(b) What are the dimensions of the Mumford-Tate domains so constructed? What is the rank of the horizontal distribution?
(c) Why doesn't (a) work for Hodge numbers $(1,2,1)$ (weight 2)?
(4) Exercise I.B.6.1 (p. 13 of I.B.6)
(5) Exercise I.B.6.2 (same page)
(6) Find a VHS to whch Prop. I.B.6.5 applies!

[1] The usual convention is to tilt this picture 45 degrees, but the "square" picture is a little easier to work with for our purposes.