FRG Meeting VIII: abstracts

Wednesday January 4

Robles (10-11): Characterization of Gross's CY variations of Hodge type by characteristic forms

Gross showed that to every Hermitian symmetric tube domain we may associate a canonical variation of Hodge structure (VHS) of Calabi-Yau type. The construction is representation theoretic, not geometric, in nature, and it is an open question to realize this abstract VHS as the variation induced by a family of polarized, algebraic Calabi-Yau manifolds.

In order for a geometric VHS to realize Gross's VHS it is necessary that the invariants associated to the two VHS coincide. For example, the Hodge numbers must agree. The later are discrete/integer invariants. Characteristic forms are differential-geometric invariants associated to VHS (introduced by Sheng and Zuo).

Remarkably, agreement of the characteristic forms is both necessary and sufficient for a geometric VHS to realize one of Gross's VHS. That is, the characteristic forms characterize Gross's Calabi-Yau VHS. I will explain this result, and discuss how characteristic forms have been used to study candidate geometric realizations of Gross's VHS.

Brosnan (11:30-12:30): Perverse obstructions to flat regular compactifications

Suppose S is a smooth, complex variety containing a dense Zariski open subset U, and suppose W is a smooth projective family of varieties over U. It seems natural to ask when W admits a regular flat compactification over S. In other words, when does there exist a smooth variety X flat and proper over S containing W as a Zariski open subset? Using resolution of singularities, it is not hard to see that it is always possible to find a regular flat compactification when S is a curve.

My main goal is to point out that, when $\dim(S) > 1$, there are obstructions coming from local intersection cohomology. My motivation is the recent preprint of Laza, Sacca and Voisin (LSV) who construct a regular flat compactification in the case that W is a certain family of abelian 5-folds over an open subset of 5 dimensional projective space. On the one hand, I'll explain how to compute the intersection cohomology in certain related examples and show that these are obstructed. On the other hand, I'll use the vanishing of the intersection cohomology obstructions implied by the LSV theorem to deduce a theorem on the palindromicity of the cohomology of certain singular cubic 3-folds.

Thursday January 5

Lian (10-11): Period integral calculus

This will be the first of three lectures on period integrals – a central object playing an intermediary role in an interplay between algebraic geometry, mirror symmetry and the theory of special functions. We begin by setting up the period sheaf of family of smooth algebraic varieties, and proceed to study this object in various settings. We mostly focus on families of Calabi-Yau hypersurfaces in a fixed ambient projective variety, but will mention generalizations to general type and Fano cases. We first consider construction of canonical sections of the 'middle degree flat bundle' associated with the family, and formulate the Riemann-Hilbert problem for the period sheaf with respect to a canonical section. We then discuss the symmetry-constraint approach and introduce a new class

of D-modules – tautological systems – as step toward the RH problem. We also discuss a number of old and new examples, and constructions in this new context. The lectures will draw from joint work with S. Bloch, J. Chen, S. Hosono, A. Huang, D. Srinivas, S.-T. Yau, X. Zhu and M. Zhu.

Harder (11:30-12:30): Hodge numbers from differential equations

I will describe how to perform explicit computations on the Hodge numbers of Calabi-Yau fibrations over the projective line by using the characteristic exponents of their Picard-Fuchs equations. I will then explain how we have used this to compute Hodge numbers of some specific $K3$-fibered Calabi-Yau threefolds and to complete some computations started by del Angel, Mueller-Stach, van Straten and Zuo. This is based on joint work with Chuck Doran and Alan Thompson.

Friday January 6

Lian (9:30-10:30): The Riemann-Hilbert problem for period integrals

We give two descriptions of a tautological systems – one algebraic and one geometric. These descriptions will be crucial to the subsequent applications to be discussed in this lecture. We apply the results to a number of old problems, including answering a question about completeness of certain extended Gel'fand-Kapranov-Zelevinsky systems and generalize it to an arbitrary projective homogeneous variety. We give a topological description of the semi-period solutions to GKZ system, answering a question of Avram et al. We also discuss a construction of 'large complex structure' degenerations of CY hypersurfaces in a few cases. We apply results on tautological systems to shed some new lights on generalized hypergeometric functions.

Huang (11-12): Zero loci of derivatives of generalized hypergeometric functions

I will explain some details regarding the rank formulas of tautological systems. As an application, I will discuss a method to compute the zero locus of derivatives of solutions to certain tautological systems: special cases include generalized hypergeometric functions, which are solutions to certain GKZ system, and certain period integrals, which are solutions to an enhanced GKZ system. This is based on joint work with J. Chen, B. Lian, and S.-T. Yau.

Saturday January 7

Lian (9:30-10:30): The hyperplane conjecture for toric hypersurfaces

In the mid 90s, Hosono et al conjectured an explicit combinatorial formula for the periods the universal family of Calabi-Yau hypersurfaces in a semi-ample toric manifold. The formula also computes the quantum cohomology as well as higher genus instantons of the mirror manifold, and has been used to systematically verify mirror symmetry in many important cases. We will discuss a recent proof of this conjectural formula. This is joint work with M. Zhu.

Zhang (11-12): A generic global Torelli theorem for certain minimal surfaces of general type

I will explain the motivations and constructions of a 16-dimensional family of minimal surfaces of general type with $p_g = 2$ and $K^2 = 1$. I will also discuss the generic global Torelli problem for

these surfaces. This talk is based on joint work with G. Pearlstein and work in progress with R. Laza and G. Pearlstein.

Sunday January 8

Harder (9:30-10:30): Picard-Fuchs equations and Shimura subvarieties

We explain, both abstractly and through concrete examples, how the problem of detecting families of lattice-polarized $K3$ surfaces with higher-than-normal Picard rank can be recast in differential algebraic terms. Starting from the uniformizing differential equation for a moduli space, we provide a complete differential algebraic characterization of the totally geodesic divisors. Rational solutions to these differential equations then correspond to rational divisors on which the Picard rank increases by one. This is joint work with Chuck Doran.

Schaffler (11-12): The KSBA compactification of the moduli space of $D_{1,6}$-polarized Enriques surfaces

In this talk we describe the moduli compactification by stable pairs (also known as KSBA compactification) of a 4-dimensional family of Enriques surfaces, which arise as the $(\mathbb{Z}_2)^2$-covers of the blow up of $\mathbb{P}^2$ at three general points branched along a configuration of three pairs of lines. The chosen divisor is an appropriate multiple of the ramification locus. Using the theory of stable toric pairs we are able to study the degenerations parametrized by the boundary and its stratification. We relate this compactification to the Baily-Borel compactification of the same family of Enriques surfaces. Part of the boundary of this stable pairs compactification has a toroidal behavior, another part is isomorphic to the Baily-Borel compactification, and what remains is a mixture of these two.