Erratum to the paper "Distinguished Varieties, Acta Math. Vol. 194"

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Lemma 4.6 is incorrect. The theorem it supports, Theorem 4.1, is correct as stated, and can be proved with a slight modification of the argument in the paper.

The error in the lemma is that there can be points of the distinguished variety

$$V \;=\; \{(z,w) \in \mathbb{D}^2 \;:\; \det[\Psi(z) - wI] = 0\}$$

where the dimension of the null space of $\Psi(z) - wI$ is discontinuous, and at these points one may not be able to choose $\hat{u}^1$ continuously.

If these singularities are disjoint from the set of nodes $\{\lambda_1, \ldots, \lambda_N\}$, then the proof of Theorem 4.1 is not materially affected. However, to include the case that a node be a singular point, we must modify the argument, and replace Lemma 4.6 with a correct version, Lemma 4.16 below.

LEMMA 4.16 *Every admissible kernel on a set $\{\lambda_1, \ldots, \lambda_N\}$ can be extended to an admissible kernel k on a distinguished variety V that contains the points $\lambda_1, \ldots, \lambda_N$. V can be represented as*

$$V \;=\; \{(z,w) \;\in\; \mathbb{D}^2 : \det[\Psi(z) - wI] = 0$$

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for some matrix-valued inner function Ψ. Moreover, the extension can be chosen in such a way that

$$k(z,w) \;=\; s_{\bar z} \otimes \hat u^1(z,w),$$

where each vector $\hat u^1(z,w)$ is in the null-space of $\det[\Psi(z) - wI]$, and so that, at each node λ_j, there are q_j sequences

$$(\alpha_m, \beta_{p,m})_{m=1}^{\infty} \qquad 1 \leq p \leq q_j$$

that converge to λ_j and such that the vector $\hat u^1(\lambda_j) = u_j^1$ is the limit of vectors in the linear span of

$$\{\hat u^1(\alpha_m, \beta_{p,m}) \;:\; 1 \leq p \leq q_j\}.$$

Proof of Lemma 4.16: Everything in the proof of Lemma 4.6 is correct except for the assertion that $(\hat u^1, \hat u^2)$ can be chosen continuously. We wish to show that these vectors can be chosen so that the conclusion of Lemma 4.16 holds.

Fix some node, λ_1 say. By Gaussian elimination, after permutation of the coordinates, there are analytic functions $f_{ij}, 1 \leq i \leq j \leq d_1$, on a neighborhood of λ_1 such that

$$\mathrm{Ker}\,(\Psi(z) - wI) \;=\; \mathrm{Ker} \begin{pmatrix} f_{11}(z,w) & f_{12}(z,w) & \dots & f_{1d_1}(z,w) \\ 0 & f_{22}(z,w) & \dots & f_{2d_1}(z,w) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & f_{d_1 d_1}(z,w) \end{pmatrix},$$

and, moreover, the diagonal functions f_{11} through f_{rr} do not vanish at λ_1, and the functions f_{jj} do vanish for $r < j \leq d_1$.

Note that the zero sets of f_{jj} are (unions of) sheets of V near λ_1. Choose any sheet of the variety near λ_1. With the exception of a possible jump at λ_1, the dimension of $\mathrm{Ker}(\Psi(z) - wI)$ will be locally constant, t say. After a permutation of the last $d_1 - r$ coordinates if necessary, we can assume the sheet corresponds to the vanishing of $f_{r+1,r+1}$ through $f_{r+t,r+t}$ (we are not assuming that they are coprime). Now choose $\hat{u}^1$ on this sheet so that its $(r+1)^{\mathrm{st}}$ through $(r+t)^{\mathrm{th}}$ coordinates agree with those of u_1^1, its $(r+t+1)^{\mathrm{th}}$ through d_1^{th} coordinates are zero, and it lies in

$$\mathrm{Ker}\begin{pmatrix} f_{11}(z,w) & f_{12}(z,w) & \dots & f_{1d_1}(z,w) \\ 0 & f_{22}(z,w) & \dots & f_{2d_1}(z,w) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & f_{d_1d_1}(z,w) \end{pmatrix}.$$

Repeat this for each sheet, and the sum of the chosen vectors will converge to u_1^1. $\square$

To prove Theorem 4.1 given Lemma 4.16, we will change the second and last paragraphs of Step 2.

We claim that at every node λ_j, the formula (4.10) uniquely defines the solution to the Pick problem on *some* sheet through λ_j. The union of all the irreducible components containing these particular sheets will give a distinguished variety containing every node on which the solution is unique.

Indeed, if the denominator in equation (4.10) vanished on every sheet of V that meets λ_j, then any linear combination of the q vectors

$$\left(w_j \widehat{K}((\alpha_m, \beta_{p,m}), \lambda_j) \right)_{j=1}^{N}$$

would be orthogonal to γ, and by taking a limit we obtain (4.11) again. The remainder of the proof proceeds as before, showing that (4.11) would contradict minimality of the problem. $\square$