REVIEW FOR EXAM III

1) What is $\lim_{n\to\infty} (1+\frac{2}{n})^n$?

solution: Look at $\lim_{n\to\infty} \ln[(1+\frac{2}{n})^n] = \lim_{n\to\infty} \frac{\ln(1+\frac{2}{n})}{\frac{1}{n}}$ (by L"Hospital's rule) = $\lim_{n\to\infty} \frac{\frac{1}{1+\frac{2}{n}}(-\frac{2}{n^2})}{-\frac{1}{n^2}} = \lim_{n\to\infty} \frac{2}{1+\frac{2}{n}} = 2$. Therefore $\lim_{n\to\infty} (1+\frac{2}{n})^n = e^2$.

2) Find the sum of $\sum_{n=1}^{\infty} 3\,(0.99)^n$, if series converges. Otherwise say it diverges.

solution: The series $\sum_{n=1}^{\infty} 3\,(0.99)^n$ is a geometric series with a = 3*0.99 and r = 0.99 < 1. This converges to $\frac{3*0.99}{1-0.99} = \frac{2.97}{0.01} = 297$.

3) Consider the series $\sum_{n=1}^{\infty} tan(\frac{1}{n^2})$. Check if it is a) absolutely convergent, b) conditionally convergent or c) divergent.

solution: Since all the terms in the sequence $\{\frac{1}{n^2}\}_{n=1}^{\infty}$ are numbers between 0 and $\frac{\pi}{2}$, all terms in the series $\sum_{n=1}^{\infty} tan(\frac{1}{n^2})$ are positive . Therefore if it converges then it converges absolutely. We can use limit comparison test for $\sum_{n=1}^{\infty} tan(\frac{1}{n^2})$ with $\sum_{n=1}^{\infty} \frac{1}{n^2}$, which converges since it is a p-series with p = 2 . $\lim_{x\to\infty} \frac{tan(\frac{1}{x^2})}{\frac{1}{x^2}}$ (by L"Hospital's rule) = $\lim_{x\to\infty} \frac{sec^2(\frac{1}{x^2})\,(-\frac{2}{x^3})}{-\frac{2}{x^3}} = \lim_{x\to\infty} sec^2(\frac{1}{x^2}) = sec^2(0) = 1 > 0$. From this it follows that $\sum_{n=1}^{\infty} tan(\frac{1}{n^2})$ is absolutely convergent.

4) Check if the series $\sum_{n=1}^{\infty} \frac{2n!}{n!\,(n+1)!}$ converges or diverges.

solution: Using the ratio test we get, $\lim_{n\to\infty} \frac{(2n+2)!}{(n+1)!\,(n+2)!} \cdot \frac{n!\,(n+1)!}{2n!} =$

$\lim_{n\to\infty} \frac{(2n+2)(2n+1)}{(n+1)(n+2)} = 4 > 0$. Therefore $\sum_{n=1}^{\infty} \frac{2n!}{n!\,(n+1)!}$ diverges.

5) Use the root test to see if the series $\sum_{n=1}^{\infty} \frac{(\ln(n))^n}{n^2}$ converges or diverges.

solution: $\sqrt[n]{\frac{(\ln(n))^n}{n^2}} = \frac{ln(n)}{n^{\frac{2}{n}}}$. Now $\lim_{n\to\infty} n^{\frac{2}{n}}$ can be calculated from $\lim_{n\to\infty} \ln(n^{\frac{2}{n}}) = \lim_{n\to\infty} \frac{2\,ln(n)}{n}$. By L"Hospital's rule the limit is zero, so

$\lim_{n\to\infty} n^{\frac{2}{n}} = e^0 = 1$. This means that $\lim_{n\to\infty} \frac{ln(n)}{n^{\frac{2}{n}}} = \infty$ and series diverges.

6) Consider the series $\sum_{n=1}^{\infty} \frac{cos(n\pi)}{n}$. Check if it is absolutely convergent,
b) conditionally convergent or c) divergent.

solution: For every positive even integer, $\cos(n\pi) = 1$ and for every odd integer, $\cos(n\pi) = -1$. So our series is the alternating series $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$. By Leibniz's theorem, since $\lim_{n\to\infty} \frac{1}{n} = 0$ and $\frac{1}{n+1} < \frac{1}{n}$, we have that $\sum_{n=1}^{\infty} \frac{cos(n\pi)}{n}$ converges.

Now if we take the absolute value for all terms we get $\sum_{n=1}^{\infty} \frac{|cos(n\pi)|}{n} = \sum_{n=1}^{\infty} \frac{1}{n}$,

which diverges, since it is a p-series with $p = 1$. Therefore $\sum_{n=1}^{\infty} \frac{cos(n\pi)}{n}$ is

conditionally convergent.