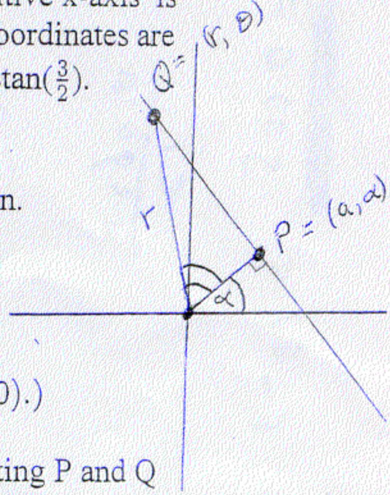
POLAR COORDINATES

1) Find a polar equation for the line through the origin having slope $\frac{3}{2}$.

Solution: We can think of that line as the line through the two points which in cartesian coordinates are (0,0) and (2 , 3). The angle that line makes with the positive x-axis is $\arctan(\frac{3}{2}) \sim 0.98$. Hence all points on that line are the points whose polar coordinates are $(r, \arctan(\frac{3}{2}))$, where r is any real number. A polar equation is $\theta = \arctan(\frac{3}{2})$.



2) Find a form for the polar equation of a line, not passing through the origin.

Solution: First find the point, P, on the line, which is closest to the origin. We'll denote the polar coordinates of that point as (a, α).

$(Draw\ the\ line\ with\ the\ point\ P,\ and\ the\ line\ segment\ from\ P\ to\ (0,0).)$

For any other point on the line, $Q = (r, \theta)$, the two line segments connecting P and Q to the origin and the line sement connecting P to Q forms a right triangle with the hypotenuse being the line from Q to the origin, it has length r. The angle between the line segments from the origin to the points P and Q is $(\theta - \alpha)$. This gives us that $\frac{a}{r} = \cos(\theta - \alpha)$, which gives us the polar equation $r = a\sec(\theta - \alpha)$.

That is, given the point closest to the origin, $P = (a, \alpha)$, the line can be described as all the points (r, θ) for which $r = a\sec(\theta - \alpha)$.

3) Find a cartesian equation for the curve whose polar equation is $r = 2a\cos(\theta)$, where a is some constant.

$r = 2a(1+\cos\theta)$

Cardioid

Solution: We have the equations $r^2 = x^2 + y^2$ and $x = r\cos(\theta)$. From this we can rewrite the equation $r = 2a\cos(\theta)$ as $\pm\sqrt{x^2+y^2} = 2a\left(\frac{x}{\pm\sqrt{x^2+y^2}}\right)$.

avoid this.

$r = 2a\cdot\cos\theta, \Leftarrow \cos\theta = \frac{x}{r}$

$\Rightarrow r^2 = 2a\cdot x \xrightarrow{r^2 = \dots} x^2+y^2 = 2ax$

We get $x^2 + y^2 = 2ax \Rightarrow x^2 - 2ax + y^2 = 0$. Completing the square we have the cartesian equation $(x-a)^2 + y^2 = a^2$ (*show the details*). We see that the curve with polar equation $r = 2a\cos(\theta)$ is the circle with radius a, having its center at the point whose cartesian coordinates are (a , 0).

OR $x^2 - 2ax + y^2 = 0$

$x^2 - 2ax + a^2 + y^2 = a^2$

$(x-a)^2 + y^2 = a^2$