1) Find $\lim_{n\to\infty} \frac{x^n}{n!}$ *where* x *is any real number.*

We can use the sandwich theorem with $-\frac{|x|}{n!} \leq \frac{x^n}{n!} \leq \frac{|x|^n}{n!}$.

If we show that $\lim_{n\to\infty} \frac{|x|^n}{n!} = 0$, *for any real number* x, *then we will have*

$\lim_{n\to\infty} \frac{x^n}{n!} = 0$. Given x, find a number M with M > |x|. Then for n > M we have

$$\frac{|x|^n}{n!} = \frac{|x|^n}{1\cdot 2\cdot 3\cdots M\cdot(M+1)\cdot(M+2)\cdots n} \leq \frac{|x|^n}{M!\cdot M^{n-M}} = \frac{|x|^n\cdot M^M}{M!\cdot M^n} = \frac{M^M}{M!}\cdot\left(\frac{|x|}{M}\right)^n.$$

Since $\frac{M^M}{M!}$ is a constant and $\frac{|x|}{M} < 1$, *we have* $\lim_{n\to\infty} \frac{|x|^n}{M!\cdot M^{n-M}} =$

$\frac{M^M}{M!} \lim_{n\to\infty} \left(\frac{|x|}{M}\right)^n = 0$. *Since* $0 \leq \frac{|x|^n}{n!} \leq \frac{|x|^n}{M!\cdot M^{n-M}}$, *by the sandwich*

theorem we have $\lim_{n\to\infty} \frac{|x|^n}{n!} = 0$ *and again by the sandwich theorem*

we have $\lim_{n\to\infty} \frac{x^n}{n!} = 0$. (Mention that this is a part of Theorem 5 in section 8.1.)

2) *Find* $\lim_{n\to\infty} \frac{n!}{n^{10}}$.

For $n > 11$ *we have* $\frac{n!}{n^{10}} = \frac{n(n-1)(n-2)\cdots(n-10)}{n^{10}} \cdot (n-11)\cdots 3\cdot 2\cdot 1 >$

$\frac{n(n-1)(n-2)\cdots(n-10)}{n^{10}} = \frac{(1-\frac{1}{n})(1-\frac{2}{n})\cdots(1-\frac{10}{n})}{\frac{1}{n}}$. By rules for limits

rules of limits (theorem 1) we get $\lim_{n\to\infty} \frac{(1-\frac{1}{n})(1-\frac{2}{n})\cdots(1-\frac{10}{n})}{\frac{1}{n}} = \frac{1}{0^+} = \infty$. Since

for n > 10 we have $\frac{n!}{n^{10}} > \frac{n(n-1)(n-2)\cdots(n-10)}{n^{10}}$ we get that $\lim_{n\to\infty} \frac{n!}{n^{10}} = \infty$.

(Tell them that they have a similar problem on this week's Webwork.)